

thm_2Ereal_topology_2ESUMMABLE_SUBSET (TMPRsqdLjxK9jhiuYiUq2SszMTGRejLkUkf)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2E_S$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ecombin_2E_C$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A_27a})).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 6 We define $c_2Ecombin_2E_o$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A_27a})).(ap V0P (ap (c_2Emin_2E_40 A$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 9 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{4}$$

Definition 13 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 14 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 17 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (5)$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Emk_net \\ A_27a \in ((ty_2Ereal_topology_2Enet\ A_27a)^{(2^{A_27a})^{A_27a}}) \end{aligned} \quad (6)$$

Definition 18 We define $c_2Ereal_topology_2Esequentially$ to be $(ap\ (c_2Ereal_topology_2Emk_net\ ty_2Ereal_topology_2Enet)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Definition 19 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 20 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (8)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (9)$$

Definition 21 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ x\ y)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (10)$$

Definition 22 We define $c_2Eiterate_2E_2E_2E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 23 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Definition 36 We define $c_2Eiterate_2ESum$ to be $\lambda A_27a : \iota.(ap (c_2Eiterate_2Eiterate A_27a ty_2Erealax_2Ereal$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (17)$$

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (18)$$

Definition 37 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Ereal_topology_2Eenetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ereal_topology_2Eenetord A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Eenet A_27a)}) \quad (19)$$

Definition 38 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Eenet A_27a)$

Definition 39 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2Ereal_topology_2Eenet A_27a)$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}) \quad (20)$$

Definition 40 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^{A_27a})$

Definition 41 We define $c_2Ereal_topology_2Esums$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}).\lambda A_27a$

Definition 42 We define $c_2Ereal_topology_2Esummable$ to be $\lambda V0s \in (2^{ty_2Eenum_2Eenum}).\lambda V1f \in (ty_2Ereal_topology_2Eenet A_27a)$

Definition 43 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epred_set_2ESUBSET A_27a s t))$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (27)$$

Assume the following.

$$(\forall V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1s \in (2^{ty_2Enum_2Enum}).(\forall V2t \in (2^{ty_2Enum_2Enum}).(\forall V3l \in ty_2Erealax_2Ereal.(((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Enum_2Enum) V1s) V2t)) \wedge (p (ap (ap (ap (c_2Ereal_topology_2Esums (\lambda V4i \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2ECOND ty_2Erealax_2Ereal) (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V4i) V1s)) (ap V0x V4i)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) V3l) V2t)))) \Rightarrow (p (ap (ap (ap c_2Ereal_topology_2Esums V0x) V3l) V1s))))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (33)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg(\\
& p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p)))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{39}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{40}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (\forall V1s \in \\
& (2^{ty_2Enum_2Enum}). (\forall V2t \in (2^{ty_2Enum_2Enum}). (((p \ (\\
& ap \ (ap \ (c_2Epred_set_2ESUBSET \ ty_2Enum_2Enum) \ V1s) \ V2t)) \wedge (p \\
& (ap \ (ap \ c_2Ereal_topology_2Esummable \ V2t) \ (\lambda V3i \in ty_2Enum_2Enum. \\
& (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ ty_2Erealax_2Ereal) \ (ap \ (ap \ (c_2Ebool_2EIN \\
& ty_2Enum_2Enum) \ V3i) \ V1s)) \ (ap \ V0x \ V3i)) \ (ap \ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))))) \Rightarrow (p \ (ap \ (ap \ c_2Ereal_topology_2Esummable \\
& \ V1s) \ V0x))))))
\end{aligned}$$