

thm_2Ereal__topology_2ETOPSPACE__EUCLIDEAN__SUBTOPOI
(TMXiuad-
vqw8YSFkeCESsJPwZhBMtDrmwNGn)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$
of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 6 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A-27b})^{A-27a}}) \quad (2)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod A_27a 2)^{A-27b}}) \quad (3)$$

Definition 8 We define $c_2Epred_set_2EINTER$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2Etopology_2Etopology : \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (4)$$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c_2Etopology_2Eopen_in A.27a \in ((2^{(2^{A-27a})})^{(ty_2Etopology_2Etopology A.27a)}) \quad (5)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c_2Etopology_2Etopology A.27a \in ((ty_2Etopology_2Etopology A.27a)^{(2^{(2^{A-27a})})}) \quad (6)$$

Definition 9 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A.27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology$

Definition 10 We define $c_2Epred_set_2EUNIV$ to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2ET)$.

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealax_2Ereal \quad (7)$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (8)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal \quad (9)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (10)$$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (the (\lambda x.x \in A \wedge P x))))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (11)$$

Definition 13 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{12}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{13}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{14}$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax\ 2Ereal^{ty_2Enum_2Enum}) \tag{15}$$

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 16 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap\ (c_2Ebool_2E_2$

Definition 17 We define $c_2Ereal_topology_2Eeuclidean$ to be $(ap\ (c_2Etopology_2Etopology\ ty_2Erealax$

Definition 18 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap\ (c_2Epred_s$

Definition 19 We define $c_2Etopology_2Etopspace$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Assume the following.

$$True \tag{16}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{17}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{18}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow ((\forall V0s \in (2^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ (c_2Epred_set_2EUNIV\ A_27a))\ V0s) = V0s)) \wedge (\forall V1s \in (2^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V1s)\ (c_2Epred_set_2EUNIV\ A_27a)) = V1s))) \tag{19}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology\ A_27a). (\forall V1u \in (2^{A_27a}). ((ap\ (c_2Etopology_2Etopspace\ A_27a)\ (ap\ (ap\ (c_2Ereal_topology_2Esubtopology\ A_27a)\ V0top)\ V1u)) = (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ (ap\ (c_2Etopology_2Etopspace\ A_27a)\ V0top))\ V1u)))) \tag{20}$$

Assume the following.

$$((ap (c_2Etopology_2Etopspace \ ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) = (c_2Epred_set_2EUNIV \ ty_2Erealax_2Ereal)) \tag{21}$$

Theorem 1

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).((ap (c_2Etopology_2Etopspace \ ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \ ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) V0s)) = V0s))$$