

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (6)$$

Definition 8 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ ty_2Erealax_2Ereal\ V0a))$

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Definition 9 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 10 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (10)$$

Definition 11 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Definition 12 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ ty_2Erealax_2Ereal\ V0P))))$

Definition 13 We define $c_2Ereal_topology_2Euniformly_continuous_on$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).\lambda V1s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2E21\ ty_2Erealax_2Ereal\ V1s))$

Definition 14 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 15 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E21))$

Definition 16 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2E21\ ty_2Erealax_2Ereal\ V0s))$

Definition 17 We define $c_2Ereal_topology_2Elimit_point_of$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1s \in ($

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (12)$$

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 19 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Definition 20 We define $c_2Ereal_topology_2Eclosure$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (ap\ (c_2Epred$

Definition 21 We define $c_2Ereal_topology_2Econtinuous_on$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Ereal})$

Definition 22 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (14)$$

Definition 23 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 24 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 25 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 26 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 27 We define $c_2Ereal_topology_2Ecauchy$ to be $\lambda V0s \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum})$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (19)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg (p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A_27a}).((\forall V2x \in A_27a.((p \ (ap \ V1P \ V2x)) \vee (p \ V0Q))) \Leftrightarrow ((\forall V3x \in A_27a.(p \ (ap \ V1P \ V3x))) \vee (p \ V0Q)))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee ((p \ V1B) \wedge (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \wedge ((p \ V0A) \vee (p \ V2C)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V1B) \wedge ((p \ V2C) \vee (p \ V0A))) \Leftrightarrow (((p \ V1B) \vee (p \ V0A)) \wedge ((p \ V2C) \vee (p \ V0A)))))) \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (28)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (29)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2s \in (2^{ty_2Erealax_2Ereal}).(((\forall V3x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V3x) V2s)) \Rightarrow ((ap V0f V3x) = (ap V1g V3x)))))) \wedge (p (ap (ap c_2Ereal_topology_2Euniformly_continuous_on V0f) V2s)) \Rightarrow (p (ap (ap c_2Ereal_topology_2Euniformly_continuous_on V1g) V2s)))))) \quad (30)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1g \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2h \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(((p (ap (ap c_2Ereal_topology_2Econtinuous_on V1g) (ap c_2Ereal_topology_2Eclosure V0s))) \wedge ((p (ap (ap c_2Ereal_topology_2Econtinuous_on V2h) (ap c_2Ereal_topology_2Eclosure V0s))) \wedge (\forall V3x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V3x) V0s)) \Rightarrow ((ap V1g V3x) = (ap V2h V3x)))))) \Rightarrow (\forall V4x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V4x) (ap c_2Ereal_topology_2Eclosure V0s))) \Rightarrow ((ap V1g V4x) = (ap V2h V4x)))))) \quad (31)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap c_2Ereal_topology_2Euniformly_continuous_on V0f) V1s)) \wedge (p (ap (ap c_2Ereal_topology_2Econtinuous_on V0f) (ap c_2Ereal_topology_2Eclosure V1s)))) \Rightarrow (p (ap (ap c_2Ereal_topology_2Euniformly_continuous_on V0f) (ap c_2Ereal_topology_2Eclosure V1s)))))) \quad (32)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}).((p (ap (ap (ap c_2Ereal_topology_2Euniformly_continuous_on \\
& V0f) V1s))) \Rightarrow (\forall V2x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). \\
& (((p (ap c_2Ereal_topology_2Ecauchy V2x)) \wedge (\forall V3n \in ty_2Enum_2Enum. \\
& (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V2x V3n)) V1s)))) \Rightarrow \\
& (p (ap (ap c_2Ereal_topology_2Ecauchy (ap (ap (c_2Ecombin_2Eo ty_2Enum_2Enum \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V2x))))))))) \\
& \hspace{15em} (33)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}).(\forall V2x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). \\
& (((p (ap c_2Ereal_topology_2Ecauchy V2x)) \wedge (\forall V3n \in ty_2Enum_2Enum. \\
& (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V2x V3n)) V1s)))) \Rightarrow \\
& (p (ap (ap c_2Ereal_topology_2Ecauchy (ap (ap (c_2Ecombin_2Eo ty_2Enum_2Enum \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V2x)))) \Rightarrow (\exists V4g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).((p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& V4g) (ap c_2Ereal_topology_2Eclosure V1s))) \wedge (\forall V5x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V5x) V1s)) \Rightarrow ((ap \\
& V4g V5x) = (ap V0f V5x))))))))) \\
& \hspace{15em} (34)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (36)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
& \hspace{15em} (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
& \hspace{15em} (38)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (39)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \wedge (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (\neg(p \vee V1q)) \vee (\neg(p \vee V2r)))) \wedge (((p \vee V1q) \vee \\
& (\neg(p \vee V0p))) \wedge ((p \vee V2r) \vee (\neg(p \vee V0p))))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \vee (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (\neg(p \vee V1q))) \wedge (((p \vee V0p) \vee (\neg(p \vee V2r))) \wedge \\
& ((p \vee V1q) \vee ((p \vee V2r) \vee (\neg(p \vee V0p))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \Rightarrow (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (p \vee V1q)) \wedge (((p \vee V0p) \vee (\neg(p \vee V2r))) \wedge (\\
& \neg(p \vee V1q)) \vee ((p \vee V2r) \vee (\neg(p \vee V0p))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee V0p) \Leftrightarrow (\neg(p \vee V1q))) \Leftrightarrow (((p \vee V0p) \vee \\
& (p \vee V1q)) \wedge ((\neg(p \vee V1q)) \vee (\neg(p \vee V0p))))))
\end{aligned} \tag{44}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow (p \vee V0p))) \tag{45}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow (\neg(p \vee V1q)))) \tag{46}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow (\neg(p \vee V0p)))) \tag{47}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow (\neg(p \vee V1q)))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \vee V0p))) \Rightarrow (p \vee V0p))) \tag{49}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}).((p (ap (ap c_2Ereal_topology_2Euniformly_continuous_on \\
& V0f) V1s)) \Rightarrow (\exists V2g \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap c_2Ereal_topology_2Euniformly_continuous_on \\
& V2g) (ap c_2Ereal_topology_2Eclosure V1s))) \wedge ((\forall V3x \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
V3x) V1s)) \Rightarrow ((ap V2g V3x) = (ap V0f V3x)))) \wedge (\forall V4h \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). \\
& (((p (ap (ap c_2Ereal_topology_2Econtinuous_on V4h) (ap c_2Ereal_topology_2Eclosure \\
& V1s))) \wedge (\forall V5x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) V5x) V1s)) \Rightarrow ((ap V4h V5x) = (ap V0f V5x)))))) \Rightarrow \\
& (\forall V6x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V6x) (ap c_2Ereal_topology_2Eclosure V1s))) \Rightarrow ((ap V4h V6x) = (\\
& ap V2g V6x))))))))))
\end{aligned}$$