

# thm\_2Ereal\_\_topology\_2EUNIFORMLY\_\_CONTINUOUS\_\_ON\_\_VM (TMQDs7JrcuoMwRQD7KDDPX4ryiMwUt2Qu89)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o(x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $c\_2Enum\_2E\_ZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2E\_ZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 3** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2E\_ZERO\_REP)$ .

**Definition 4** We define  $c\_2Earithmetic\_2E\_ZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2E\_REP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2E\_REP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2E\_SUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2E\_SUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 6** We define  $c\_2Enum\_2E\_SUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ ($

Let  $c\_2Earithmic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 7** We define  $c\_2Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmic\_2E\_2B))$

**Definition 8** We define  $c\_2Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (7)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (8)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (9)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (10)$$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap\ P\ x))$  **then** *(the  $(\lambda x.x \in A \wedge p)$  of type  $\iota \Rightarrow \iota$ ).*

**Definition 10** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ a))$

Let  $c\_2Erealax\_2Etreall\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (11)$$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Erealax\_2Etreall\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (13)$$

Let  $c\_2Erealax\_2Etreall\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (14)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})}) \quad (15)$$

**Definition 12** We define  $c\_Erealax\_Ereal\_ABS$  to be  $\lambda V0r \in (ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)$

**Definition 13** We define  $c\_Erealax\_Ereal\_neg$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.(ap\ c\_Erealax\_Ereal)$

**Definition 14** We define  $c\_Ebool\_EF$  to be  $(ap\ (c\_Ebool\_E21\ 2))\ (\lambda V0t \in 2.V0t)$ .

**Definition 15** We define  $c\_Emin\_E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 16** We define  $c\_Ebool\_E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_Emin\_E3D\_3D\_3E\ V0t)\ c\_Ebool\_E7E))$

**Definition 17** We define  $c\_Ereal\_Ereal\_lte$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal$

Let  $c\_Erealax\_Etrealmul : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealmul \in (((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal)}}_{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal)} \quad (16)$$

**Definition 18** We define  $c\_Erealax\_Ereal\_mul$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal$

Let  $c\_Erealax\_Etrealmul : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealmul \in (((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal)}}_{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal)} \quad (17)$$

**Definition 19** We define  $c\_Erealax\_Ereal\_add$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal$

**Definition 20** We define  $c\_Ebool\_E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_Ebool\_E21\ 2))\ (\lambda V2t \in 2.V2t)))$

**Definition 21** We define  $c\_Ereal\_topology\_Elinear$  to be  $\lambda V0f \in (ty\_Erealax\_Ereal^{ty\_Erealax\_Ereal})$

**Definition 22** We define  $c\_Ecombin\_Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g \in (A\_27a^{A\_27c}).$

**Definition 23** We define  $c\_Ebool\_EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Let  $c\_Epair\_EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_Epair\_EABS\_prod\ A\_27a\ A\_27b \in ((ty\_Epair\_Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (18)$$

**Definition 24** We define  $c\_Epair\_E2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_Epair\_EABS\_prod\ A\_27a\ A\_27b)\ V0x\ V1y)$

Let  $c\_Epred\_set\_EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_Epred\_set\_EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_Epair\_Eprod\ A\_27a\ 2)^{A\_27b}}) \quad (19)$$

**Definition 25** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$   
Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (20)$$

**Definition 26** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 27** We define  $c\_2Ereal\_topology\_2Euniformly\_continuous\_on$  to be  
 $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).\lambda V1s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (c\_2Ebool\_2E\_21\ ty\_2Ereal$

**Definition 28** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$   
Assume the following.

$$True \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p\ V0t)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in \\ & (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A\_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A\_27a. (p\ ( \\ & \quad ap\ V1Q\ V4x))))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee ( \\ & (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg( \\ & p\ V0A) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge (\neg(p\ V1B)))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \\ & ((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0r \in 2. (\forall V1p \in 2. (\forall V2q \in 2. (((p\ V1p) \wedge \\ & (p\ V2q) \Rightarrow (p\ V0r)) \Leftrightarrow ((p\ V1p) \Rightarrow ((p\ V2q) \Rightarrow (p\ V0r)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\ & ((ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V0x)\ V1y) = (ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & \quad V1y)\ V0x)))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\ & (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & V0x)\ (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V1y)\ V2z)) = (ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & \quad (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V0x)\ V1y))\ V2z)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & (ap\ c\_2Erealax\_2Ereal\_neg\ V0x)\ V0x) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\ & \quad c\_2Enum\_2E0))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& \quad V1y) V0x))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& V0x) (ap (ap c\_2Erealax\_2Ereal\_mul V1y) V2z)) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& \quad (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)) V2z))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& \quad ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
& \quad V0x) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = V0x))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) (ap c\_2Erealax\_2Ereal\_neg V0x)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_add V0x) \\
& V1y)) = (ap (ap c\_2Erealax\_2Ereal\_add (ap c\_2Erealax\_2Ereal\_neg \\
& \quad V0x)) (ap c\_2Erealax\_2Ereal\_neg V1y))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V0x) V0x)))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1y) V0x)))) \Leftrightarrow (V0x = V1y)))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& V1y) = (ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V0x) V1y))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty\_2Erealax\_2Ereal. (\forall V1x \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V0y) V1x)))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& V1y)) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0) (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& (ap c\_2Erealax\_2Ereal\_neg V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1y) V0x)))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg \\
& (ap c\_2Erealax\_2Ereal\_neg V0x)) = V0x))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) (ap c\_2Erealax\_2Ereal\_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) V1y)) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) V2z) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V2z)) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V1y) V2z)))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).((p (ap c\_2Ereal\_topology\_2Elinear \\
& \quad V0f)) \Rightarrow (p (ap (ap c\_2Ereal\_topology\_2Euniformly\_continuous\_on \\
& \quad \quad V0f) V1s)))))) \quad (53)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2s \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad \quad ((p (ap (ap c\_2Ereal\_topology\_2Euniformly\_continuous\_on \\
& \quad \quad V0f) V2s)) \wedge (p (ap (ap c\_2Ereal\_topology\_2Euniformly\_continuous\_on \\
& \quad V1g) (ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad \quad V0f) V2s)))) \Rightarrow (p (ap (ap c\_2Ereal\_topology\_2Euniformly\_continuous\_on \\
& \quad \quad (ap (ap (c\_2Ecombin\_2Eo ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal \\
& \quad \quad \quad ty\_2Erealax\_2Ereal) V1g) V0f)) V2s)))))) \quad (54)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (56)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (57)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (58)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (59)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\
& \quad (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& \quad p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& \quad \quad ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (60)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\
& \quad (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& \quad \quad (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (61)
\end{aligned}$$



Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee \neg(p V1q)) \wedge ((p V0p) \vee \neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p))))))
\end{aligned} \tag{63}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}). (\forall V1c \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& (\forall V2v \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Ereal\_topology\_2Euniformly\_continuous\_on \\
& V1c) V0s)) \Rightarrow (p (ap (ap c\_2Ereal\_topology\_2Euniformly\_continuous\_on \\
& (\lambda V3x \in ty\_2Erealax\_2Ereal. (ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap V1c V3x)) V2v))) V0s))))))
\end{aligned}$$