

thm_2Ereal__topology_2EURYSOHN__STRONG (TMYQUsaGRkryGu4Mxbxbydoqm27Hf9NkMUU)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (1)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealax_2Ereal \quad (2)$$

Definition 6 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (3)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (4)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epred_set_2EGSPEC A.27a A.27b \in ((2^{A.27a})^{(ty_2Epair_2Eprod A.27a 2)^{A.27b}})$$
(5)

Definition 11 We define $c_2Epred_set_2EDIFF$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A.27a}).\lambda V1t \in (2^{A.27a}).(ap (c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)})$$
(6)

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal$$
(7)

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal})$$
(8)

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)})$$
(9)

Definition 14 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
(10)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum$$
(11)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(12)

Definition 15 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum})$$
(13)

Definition 16 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_Emin_2E_40$

Definition 17 We define $c_Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap (c_Ebool_2E_2$

Definition 18 We define $c_Ereal_topology_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap c_Ereal_topo$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (14)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (15)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (16)$$

Definition 19 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (17)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (18)$$

Definition 20 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (19)$$

Definition 21 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic$

Definition 22 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 23 We define $c_Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Er$

Let $c_2Elist_2EHHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EHHD A_27a \in (A_27a)^{(ty_2Elist_2Elist A_27a)} \quad (20)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (21)$$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (22)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (23)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}}) \quad (24)$$

Definition 24 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 25 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (25)$$

Let $c_2Erealax_2Etreal_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (26)$$

Definition 26 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Erealax_2Etreal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (27)$$

Definition 27 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 28 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 29 We define $c_2Ereal_topology_2Eclosed_segment$ to be $\lambda V0l \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))$

Definition 30 We define $c_2Ereal_topology_2Econtinuous_on$ to be $\lambda V0f \in (ty_2Erealax_2Ereal)^{ty_2Erealax_2Ereal}$

Definition 31 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 32 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Erealax_2Ereal_mul\ s\ t))$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^A-27a)})}) \quad (28)$$

Definition 33 We define $c_2Ereal_topology_2Euclidean$ to be $(ap\ (c_2Etopology_2Etopology\ ty_2Erealax$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Eopen_in\ A_27a \in ((2^{(2^A-27a)})^{(ty_2Etopology_2Etopology\ A_27a)}) \quad (29)$$

Definition 34 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology$

Definition 35 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^A-27a)}).(ap\ (c_2Epred_set$

Definition 36 We define $c_2Etopology_2Etopspace$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology$

Definition 37 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^A-27a).\lambda V1t \in (2^A-27a).(ap\ ($

Definition 38 We define $c_2Etopology_2Eclosed_in$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology$

Assume the following.

$$True \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EUNIV\ A_27a)))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology\ A_27a).((ap\ (ap\ (c_2Ereal_topology_2Esubtopology\ A_27a)\ V0top)\ (c_2Epred_set_2EUNIV\ A_27a)) = V0top)) \quad (35)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2EClosed V0s)) \Leftrightarrow (p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) V0s)))) \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\ & (\forall V2u \in (2^{ty_2Erealax_2Ereal}).(\forall V3a \in ty_2Erealax_2Ereal. \\ & (\forall V4b \in ty_2Erealax_2Ereal.(((p (ap (ap (c_2Etopology_2Eclosed_in \\ & ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\ & ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) V2u)) \\ & V0s)) \wedge ((p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) \\ & (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\ & c_2Ereal_topology_2Euclidean) V2u)) V1t) \wedge ((ap (ap (c_2Epred_set_2EINTER \\ & ty_2Erealax_2Ereal) V0s) V1t) = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)) \wedge \\ & (\neg(V3a = V4b)))))) \Rightarrow (\exists V5f \in (ty_2Erealax_2Ereal)^{ty_2Erealax_2Ereal}). \\ & ((p (ap (ap c_2Ereal_topology_2Econtinuous_on V5f) V2u)) \wedge \\ & (\forall V6x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ & V6x) V2u)) \Rightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V5f \\ & V6x)) (ap c_2Ereal_topology_2EClosed_segment (ap (ap (c_2Elist_2ECONS \\ & (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\ & (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\ & V3a) V4b)) (c_2Elist_2ENIL (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal)))))))))) \wedge ((\forall V7x \in ty_2Erealax_2Ereal. \\ & ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V7x) V2u)) \Rightarrow (((\\ & ap V5f V7x) = V3a) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ & V7x) V0s)))))) \wedge (\forall V8x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN \\ & ty_2Erealax_2Ereal) V8x) V2u)) \Rightarrow (((ap V5f V8x) = V4b) \Leftrightarrow (p (ap (ap \\ & (c_2Ebool_2EIN ty_2Erealax_2Ereal) V8x) V1t))))))))))))) \quad (37) \end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V2a \in ty_2Erealax_2Ereal. (\forall V3b \in ty_2Erealax_2Ereal. \\
& (((p (ap c_2Ereal_topology_2EClosed V0s)) \wedge ((p (ap c_2Ereal_topology_2EClosed \\
& V1t)) \wedge (((ap (ap (c_2Epred_set_2EINTER ty_2Erealax_2Ereal) \\
& V0s) V1t) = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)) \wedge (\neg(\\
& V2a = V3b)))))) \Rightarrow (\exists V4f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap c_2Ereal_topology_2Econtinuous_on V4f) (c_2Epred_set_2EUNIV \\
& ty_2Erealax_2Ereal)))) \wedge ((\forall V5x \in ty_2Erealax_2Ereal. (\\
p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V4f V5x)) (ap c_2Ereal_topology_2EClosed_segment \\
& (ap (ap (c_2Elist_2ECONS (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)) (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V2a) V3b)) (c_2Elist_2ENIL (ty_2Epair_2Eprod \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal)))))) \wedge ((\forall V6x \in \\
& ty_2Erealax_2Ereal. (((ap V4f V6x) = V2a) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) V6x) V0s)))) \wedge (\forall V7x \in ty_2Erealax_2Ereal. \\
& (((ap V4f V7x) = V3b) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V7x) V1t))))))))))
\end{aligned}$$