

thm_2Ereal_topology_2Einterval
(TMFn7zFQgX9Z9LyBnN5eiB9Z96YbP5GPetn)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ENIL\ A.27a \in (ty_2Elist_2Elist\ A.27a) \quad (2)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$.

Definition 3 We define c_2Ebool_2E21 to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a})))$

Definition 4 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ECONS\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})^{A.27a}) \quad (3)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (4)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (5)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (6)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (7)$$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 6 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (ty_2Erealax\ V0a)))$

Let $c_2Erealax_2Etreall_It : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_It \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Erealax)}) \quad (8)$$

Definition 7 We define $c_2Erealax_2Ereal_It$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(c_2Etreall_It\ V0T1\ V1T2)$

Let $c_2Ereal_topology_2EOPEN_interval : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EOPEN_interval \in ((2^{(ty_2Erealax_2Ereal)})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Ereal)}) \quad (9)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (10)$$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EHD\ A_27a \in (A_27a)^{(ty_2Elist_2Elist\ A_27a)} \quad (11)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (12)$$

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Definition 10 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.(c_2Erealax_2Ereal\ V0x\ V1y)$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.(c_2Ebool_2E_21\ 2)\ V2t)\ V1t2)\ V0t1))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (13)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod A_27a 2)^{A_27b})}) \end{aligned} \quad (14)$$

Definition 13 We define $c_2Ereal_topology_2ECLOSED_interval$ to be $\lambda V0l \in (ty_2Elist_2Elist (ty_2Epa$ Assume the following.

$$True \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow (\forall V0h \in A_27a.(\forall V1t \in \\ (ty_2Elist_2Elist A_27a).((ap (c_2Elist_2EHD A_27a) (ap (ap (\\ c_2Elist_2ECONS A_27a) V0h) V1t)) = V0h))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ \forall V0x \in A_27a.(\forall V1y \in A_27b.((ap (c_2Epair_2EFST A_27a \\ A_27b) (ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y)) = V0x))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ \forall V0x \in A_27a.(\forall V1y \in A_27b.((ap (c_2Epair_2ESND A_27a \\ A_27b) (ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y)) = V1y))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} (\forall V0a \in ty_2Erealx_2Ereal.(\forall V1b \in ty_2Erealx_2Ereal. \\ ((ap c_2Ereal_topology_2EOPEN_interval (ap (ap (c_2Epair_2E_2C \\ ty_2Erealx_2Ereal ty_2Erealx_2Ereal) V0a) V1b)) = (ap (c_2Epred_set_2EGSPEC \\ ty_2Erealx_2Ereal ty_2Erealx_2Ereal) (\lambda V2x \in ty_2Erealx_2Ereal. \\ (ap (ap (c_2Epair_2E_2C ty_2Erealx_2Ereal 2) V2x) (ap (ap c_2Ebool_2E_2F_5C \\ (ap (ap c_2Erealx_2Ereal_lt V0a) V2x)) (ap (ap c_2Erealx_2Ereal_lt \\ V2x) V1b)))))))))) \end{aligned} \quad (21)$$

Theorem 1

$$\begin{aligned} & (\forall V0a \in ty_2Erealax_2Ereal. (\forall V1b \in ty_2Erealax_2Ereal. \\ & (((ap\ c_2Ereal_topology_2EOPEN_interval\ (ap\ (ap\ (c_2Epair_2E_2C \\ & ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V0a)\ V1b)) = (ap\ (c_2Epred_set_2EGSPEC \\ & ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ (\lambda V2x \in ty_2Erealax_2Ereal. \\ & (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ 2)\ V2x)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\ & (ap\ (ap\ c_2Erealax_2Ereal_lt\ V0a)\ V2x))\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\ & V2x)\ V1b)))))) \wedge ((ap\ c_2Ereal_topology_2ECLOSED_interval \\ & (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal))\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal)\ V0a)\ V1b))\ (c_2Elist_2ENIL\ (ty_2Epair_2Eprod \\ & ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)))) = (ap\ (c_2Epred_set_2EGSPEC \\ & ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ (\lambda V3x \in ty_2Erealax_2Ereal. \\ & (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ 2)\ V3x)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\ & (ap\ (ap\ c_2Ereal_2Ereal_lte\ V0a)\ V3x))\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\ & V3x)\ V1b)))))))))) \end{aligned}$$