

thm\_2Erealax\_2EHREAL\_LT\_ADD2  
(TMY2Myw7zfZbU7dQSwRjRTSo134Kfq85D7k)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 5** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t$

Let  $ty\_2Ehrat\_2Ehrat : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehrat\_2Ehrat \tag{1}$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{2}$$

Let  $c\_2Ehreal\_2Ecut : \iota$  be given. Assume the following.

$$c\_2Ehreal\_2Ecut \in ((2^{ty\_2Ehrat\_2Ehrat})^{ty\_2Ehreal\_2Ehreal}) \tag{3}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{4}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{5}$$

Let  $c\_2Ehrat\_2Ehrat\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehreal}) \tag{6}$$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p (ap P x))$  **then**  $(the (\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define  $c\_2Ehrat\_2Ehrat\_REP$  to be  $\lambda V0a \in ty\_2Ehrat\_2Ehrat.(ap (c\_2Emin\_2E\_40 (ty\_2E$   
Let  $c\_2Ehrat\_2Etrat\_add : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_add \in (((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)})) \quad (7)$$

Let  $c\_2Ehrat\_2Etrat\_eq : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)}) \quad (8)$$

Let  $c\_2Ehrat\_2Ehrat\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_ABS\_CLASS \in (ty\_2Ehrat\_2Ehrat^{(2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})}) \quad (9)$$

**Definition 10** We define  $c\_2Ehrat\_2Ehrat\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2$

**Definition 11** We define  $c\_2Ehrat\_2Ehrat\_add$  to be  $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2E$

**Definition 12** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

Let  $c\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$c\_2Ehreal\_2Ehreal \in (ty\_2Ehreal\_2Ehreal^{(2^{ty\_2Ehrat\_2Ehrat})}) \quad (10)$$

**Definition 13** We define  $c\_2Ehreal\_2Ehreal\_add$  to be  $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2$

**Definition 14** We define  $c\_2Ehreal\_2Ehreal\_lt$  to be  $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2E$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal. ((ap (ap c\_2Ehreal\_2Ehreal\_add V0X) V1Y) = (ap (ap c\_2Ehreal\_2Ehreal\_add V1Y) V0X)))) \quad (13)$$

Assume the following.

$$\begin{aligned}
& (\forall V0X \in ty\_2Ehreal\_2Ehreal. (\forall V1Y \in ty\_2Ehreal\_2Ehreal. \\
& (\forall V2Z \in ty\_2Ehreal\_2Ehreal. ((ap (ap c\_2Ehreal\_2Ehreal\_add \\
V0X) (ap (ap c\_2Ehreal\_2Ehreal\_add V1Y) V2Z)) = (ap (ap c\_2Ehreal\_2Ehreal\_add \\
& (ap (ap c\_2Ehreal\_2Ehreal\_add V0X) V1Y)) V2Z))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0X \in ty\_2Ehreal\_2Ehreal. (\forall V1Y \in ty\_2Ehreal\_2Ehreal. \\
& ((p (ap (ap c\_2Ehreal\_2Ehreal\_lt V0X) V1Y)) \Leftrightarrow (\exists V2D \in ty\_2Ehreal\_2Ehreal. \\
& (V1Y = (ap (ap c\_2Ehreal\_2Ehreal\_add V0X) V2D))))))
\end{aligned} \tag{15}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0x1 \in ty\_2Ehreal\_2Ehreal. (\forall V1x2 \in ty\_2Ehreal\_2Ehreal. \\
& (\forall V2y1 \in ty\_2Ehreal\_2Ehreal. (\forall V3y2 \in ty\_2Ehreal\_2Ehreal. \\
& (((p (ap (ap c\_2Ehreal\_2Ehreal\_lt V0x1) V2y1)) \wedge (p (ap (ap c\_2Ehreal\_2Ehreal\_lt \\
V1x2) V3y2))) \Rightarrow (p (ap (ap c\_2Ehreal\_2Ehreal\_lt (ap (ap c\_2Ehreal\_2Ehreal\_add \\
& V0x1) V1x2)) (ap (ap c\_2Ehreal\_2Ehreal\_add V2y1) V3y2))))))
\end{aligned}$$