

thm\_2Erealax\_2EHREAL\_LT\_NE  
(TMG7aPAZf4tggBSsVbuKz2xz4vcNBHoLH6m)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 3** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

Let  $ty\_2Ehrrat\_2Ehrrat : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehrrat\_2Ehrrat \tag{1}$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{2}$$

Let  $c\_2Ehreal\_2Ecut : \iota$  be given. Assume the following.

$$c\_2Ehreal\_2Ecut \in ((2^{ty\_2Ehrrat\_2Ehrrat})^{ty\_2Ehreal\_2Ehreal}) \tag{3}$$

**Definition 5** We define  $c\_2Ebool\_2E\_2E$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 7** We define  $c\_2Ebool\_2E\_2E$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define  $c\_2Ebool\_2E\_2E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2E))$

**Definition 9** We define  $c\_2Ebool\_2E\_2E\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 10** We define  $c\_2Ehreal\_2Ehreal\_lt$  to be  $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2Ehreal$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (4)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (5)$$

Let  $c\_2Ehrat\_2Ehrat\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehrat}) \quad (6)$$

**Definition 11** We define  $c\_2Ehrat\_2Ehrat\_REP$  to be  $\lambda V0a \in ty\_2Ehrat\_2Ehrat.(ap\ (c\_2Emin\_2E40\ (ty\_2Ehrat\_2Ehrat)))$

Let  $c\_2Ehrat\_2Etrat\_add : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (7)$$

Let  $c\_2Ehrat\_2Etrat\_eq : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (8)$$

Let  $c\_2Ehrat\_2Ehrat\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_ABS\_CLASS \in (ty\_2Ehrat\_2Ehrat^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}) \quad (9)$$

**Definition 12** We define  $c\_2Ehrat\_2Ehrat\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$

**Definition 13** We define  $c\_2Ehrat\_2Ehrat\_add$  to be  $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2Ehrat$

Let  $c\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$c\_2Ehreal\_2Ehreal \in (ty\_2Ehreal\_2Ehreal^{(2^{ty\_2Ehrat\_2Ehrat})}) \quad (10)$$

**Definition 14** We define  $c\_2Ehreal\_2Ehreal\_add$  to be  $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2Ehreal$

Assume the following.

$$\begin{aligned} & (\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal. \\ & ((p\ (ap\ (ap\ c\_2Ehreal\_2Ehreal\_lt\ V0X)\ V1Y)) \Leftrightarrow (\exists V2D \in ty\_2Ehreal\_2Ehreal. \\ & (V1Y = (ap\ (ap\ c\_2Ehreal\_2Ehreal\_add\ V0X)\ V2D)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Ehreal\_2Ehreal.(\forall V1y \in ty\_2Ehreal\_2Ehreal. \\ & (\neg(V0x = (ap\ (ap\ c\_2Ehreal\_2Ehreal\_add\ V0x)\ V1y)))) \end{aligned} \quad (12)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0x \in ty\_2Ehreal\_2Ehreal.(\forall V1y \in ty\_2Ehreal\_2Ehreal. \\ & ((p\ (ap\ (ap\ c\_2Ehreal\_2Ehreal\_lt\ V0x)\ V1y)) \Rightarrow (\neg(V0x = V1y)))) \end{aligned}$$