

thm\_2Erealax\_2EREAL\_ISO\_EQ  
(TMX8aA4V5cWJ3dLAAHNdjTXzwxNZGEKkoEq)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 7** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 8** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 9** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a}) A\_27a$

**Definition 10** We define  $c\_2Equotient\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 11** We define  $c\_2Equotient\_2E\_3D\_3D\_3D\_3E$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27b})^{A\_27a}$

**Definition 12** We define  $c\_2Ecombin\_2EW$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x \in A\_27a.(\lambda V2y \in A\_27b.V0f (V1x V2y)))$

**Definition 13** We define  $c\_2Equotient\_2Erespects$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(c\_2Ecombin\_2EW A\_27a A\_27a A\_27b)$

**Definition 14** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 15** We define  $c\_2Ebool\_2ERES\_FORALL$  to be  $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).(ap V1m V0p)))$

**Definition 16** We define  $c\_2Equotient\_2EEQUIV$  to be  $\lambda A\_27a : \iota.\lambda V0E \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2EIN V0E))$

**Definition 17** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EIN))$

Let  $ty\_2Ehtrat\_2Ehtrat : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehtrat\_2Ehtrat \quad (1)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (2)$$

Let  $c\_2Ehreal\_2Ecut : \iota$  be given. Assume the following.

$$c\_2Ehreal\_2Ecut \in ((2^{ty\_2Ehtrat\_2Ehtrat})^{ty\_2Ehreal\_2Ehreal}) \quad (3)$$

**Definition 18** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 19** We define  $c\_2Ehreal\_2Ehreal\_lt$  to be  $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2Ehreal$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (4)$$

Let  $c\_2Erealax\_2Etreall\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (5)$$

**Definition 20** We define  $c\_2Equotient\_2EQUOTIENT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (7)$$

**Definition 21** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p\ x)\ \mathbf{of\ type}\ \iota \Rightarrow \iota).$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ (the\ (\lambda x.x \in A \wedge p\ x)\ \mathbf{of\ type}\ \iota \Rightarrow \iota))))$

Let  $c\_2Erealax\_2Etreall\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (8)$$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{9}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{10}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{11}$$

**Definition 24** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \tag{12}$$

**Definition 25** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ x\ y)$

**Definition 26** We define  $c\_2Ehrat\_2Etrat\_1$  to be  $(ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)\ ty\_2Enum\_2Enum)$

Let  $c\_2Ehrat\_2Etrat\_eq : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \tag{13}$$

Let  $c\_2Ehrat\_2Ehrat\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_ABS\_CLASS \in (ty\_2Ehrat\_2Ehrat^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}) \tag{14}$$

**Definition 27** We define  $c\_2Ehrat\_2Ehrat\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$

**Definition 28** We define  $c\_2Ehrat\_2Ehrat\_1$  to be  $(ap\ c\_2Ehrat\_2Ehrat\_ABS\ c\_2Ehrat\_2Etrat\_1)$ .

Let  $c\_2Ehrat\_2Ehrat\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehrat\_1}) \tag{15}$$

**Definition 29** We define  $c\_2Ehrat\_2Ehrat\_REP$  to be  $\lambda V0a \in ty\_2Ehrat\_2Ehrat.(ap\ (c\_2Emin\_2E.40\ (ty\_2Ehrat\_2Ehrat\_1\ a))\ a)$

Let  $c\_2Ehrat\_2Etrat\_add : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum})^{ty\_2Ehrat\_2Ehrat\_1}) \tag{16}$$

**Definition 30** We define  $c\_2Ehrat\_2Ehrat\_add$  to be  $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2Ehrat.(ap\ (c\_2Emin\_2E.40\ (ty\_2Ehrat\_2Ehrat\_1\ T1)\ T2))\ T1\ T2$

**Definition 31** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 32** We define  $c\_2Ehreal\_2Ehrat\_lt$  to be  $\lambda V0x \in ty\_2Ehrat\_2Ehreal. \lambda V1y \in ty\_2Ehrat\_2Ehreal.$

**Definition 33** We define  $c\_2Ehreal\_2Ecut\_of\_hrat$  to be  $\lambda V0x \in ty\_2Ehrat\_2Ehreal. (\lambda V1y \in ty\_2Ehrat\_2Ehreal.$

Let  $c\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$c\_2Ehreal\_2Ehreal \in (ty\_2Ehreal\_2Ehreal^{(2^{ty\_2Ehreal\_2Ehreal})}) \quad (17)$$

**Definition 34** We define  $c\_2Ehreal\_2Ehreal\_1$  to be  $(ap\ c\_2Ehreal\_2Ehreal\ (ap\ c\_2Ehreal\_2Ecut\_of\_hrat$

**Definition 35** We define  $c\_2Ehreal\_2Ehreal\_add$  to be  $\lambda V0X \in ty\_2Ehreal\_2Ehreal. \lambda V1Y \in ty\_2Ehreal\_2Ehreal.$

**Definition 36** We define  $c\_2Erealax\_2Etreax\_of\_hreal$  to be  $\lambda V0x \in ty\_2Ehreal\_2Ehreal. (ap\ (ap\ (c\_2Epair$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})}) \quad (18)$$

**Definition 37** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal$

**Definition 38** We define  $c\_2Erealax\_2Ereal\_of\_hreal$  to be  $\lambda V0T1 \in ty\_2Ehreal\_2Ehreal. (ap\ c\_2Erealax\_2Ereal$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow \neg (p\ V0t)))))) \quad (22)$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg (p\ V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (25)$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap (c\_2Ecombin\_2EI A\_27a) V0x) = V0x)) \quad (26)$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal. (\forall V1Y \in ty\_2Ehreal\_2Ehreal. ((V0X = V1Y) \vee ((p (ap (ap c\_2Ehreal\_2Ehreal\_lt V0X) V1Y)) \vee (ap (ap c\_2Ehreal\_2Ehreal\_lt V1Y) V0X)))))) \quad (27)$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow (p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT A\_27a A\_27a) (c\_2Emin\_2E\_3D A\_27a)) (c\_2Ecombin\_2EI A\_27a)) (c\_2Ecombin\_2EI A\_27a))) \quad (28)$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). ((\forall V1x \in A\_27a. (\forall V2y \in A\_27a. ((p (ap (ap V0R V1x) V2y)) \Leftrightarrow ((ap V0R V1x) = (ap V0R V2y)))))) \Leftrightarrow ((\forall V3x \in A\_27a. (p (ap (ap V0R V3x) V3x))) \wedge ((\forall V4x \in A\_27a. (\forall V5y \in A\_27a. ((p (ap (ap V0R V4x) V5y)) \Rightarrow (p (ap (ap V0R V5y) V4x)))))) \wedge (\forall V6x \in A\_27a. (\forall V7y \in A\_27a. (\forall V8z \in A\_27a. (((p (ap (ap V0R V6x) V7y)) \wedge (p (ap (ap V0R V7y) V8z))) \Rightarrow (p (ap (ap V0R V6x) V8z)))))))))) \quad (29)$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow \forall A\_27b. \text{nonempty } A\_27b \Rightarrow \forall A\_27c. \text{nonempty } A\_27c \Rightarrow \forall A\_27d. \text{nonempty } A\_27d \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}). (\forall V1abs1 \in (A\_27c^{A\_27a}). (\forall V2rep1 \in (A\_27a^{A\_27c}). ((p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT A\_27a A\_27c) V0R1) V1abs1) V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}). (\forall V4abs2 \in (A\_27d^{A\_27b}). (\forall V5rep2 \in (A\_27b^{A\_27d}). ((p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT A\_27b A\_27d) V3R2) V4abs2) V5rep2)) \Rightarrow (p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT (A\_27b^{A\_27a}) (A\_27d^{A\_27c})) (ap (ap (c\_2Equotient\_2E\_3D\_3D\_3D\_3E A\_27a A\_27b) V0R1) V3R2)) (ap (ap (c\_2Equotient\_2E\_2D\_2D\_3E A\_27c A\_27b A\_27a A\_27d) V2rep1) V4abs2)) (ap (ap (c\_2Equotient\_2E\_2D\_2D\_3E A\_27a A\_27d A\_27c A\_27b) V1abs1) V5rep2)))))))))) \quad (30)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\
& \quad (2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\
& \quad (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in \\
& \quad (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27d^{A\_27c}). \\
& \quad ((\lambda V7x \in A\_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E \\
& \quad A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A\_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x))))))))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0REL \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& \quad (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A\_27a.(\forall V4x2 \in \\
& \quad A\_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2))))))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& \quad (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27b}).((p\ ( \\
& \quad ap\ (c\_2Ebool\_2E\_21\ A\_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL \\
& \quad A\_27a)\ (ap\ (c\_2Equotient\_2Erespects\ A\_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c\_2Equotient\_2E\_2D\_2D\_3E\ A\_27a\ 2\ A\_27b\ 2)\ V1abs)\ (c\_2Ecombin\_2EI \\
& \quad 2))\ V3f))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& \quad (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27a}).(\forall V4g \in \\
& \quad (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E\ A\_27a \\
& \quad 2)\ V0R)\ (c\_2Emin\_2E\_3D\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL \\
& \quad A\_27a)\ (ap\ (c\_2Equotient\_2Erespects\ A\_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ ( \\
& \quad ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL\ A\_27a)\ (ap\ (c\_2Equotient\_2Erespects \\
& \quad A\_27a\ 2)\ V0R))\ V4g))))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\
& \quad (2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\
& \quad (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in \\
& \quad (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27b^{A\_27a}). \\
& \quad (\forall V7g \in (A\_27b^{A\_27a}).(\forall V8x \in A\_27a.(\forall V9y \in \\
& \quad A\_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E\ A\_27a \\
& \quad A\_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ ( \\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y)))))))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0E \in ((2^{A\_27a})^{A\_27a}). \\
& \quad (\forall V1P \in (2^{A\_27a}).((p\ (ap\ (c\_2Equotient\_2EEQUIV\ A\_27a) \\
& \quad V0E)) \Rightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL\ A\_27a)\ (ap\ (c\_2Equotient\_2Erespects \\
& \quad A\_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c\_2Ebool\_2E\_21\ A\_27a)\ V1P))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad (\forall V1q \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad ((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_eq\ V0p)\ V1q)) \Leftrightarrow ((ap\ c\_2Erealax\_2Etreal\_eq \\
& \quad V0p) = (ap\ c\_2Erealax\_2Etreal\_eq\ V1q))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad (\neg(p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_lt\ V0x)\ V0x))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad (\forall V1y \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad (\forall V2z \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad (((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_lt\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_lt \\
& \quad V1y)\ V2z))) \Rightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_lt\ V0x)\ V2z))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty\_2Ehreal\_2Ehreal.(\forall V1i \in ty\_2Ehreal\_2Ehreal. \\
& \quad ((p\ (ap\ (ap\ c\_2Ehreal\_2Ehreal\_lt\ V0h)\ V1i)) \Rightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_lt \\
& \quad (ap\ c\_2Erealax\_2Etreal\_of\_hreal\ V0h))\ (ap\ c\_2Erealax\_2Etreal\_of\_hreal \\
& \quad V1i))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& (\forall V1x2 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& (\forall V2y1 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& (\forall V3y2 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& (((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_eq \\
& \quad V2y1)\ V3y2))) \Rightarrow ((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_lt\ V0x1)\ V2y1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_lt\ V1x2)\ V3y2))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\
& \quad ty\_2Ehreal\_2Ehreal)\ ty\_2Erealax\_2Ereal)\ c\_2Erealax\_2Etreal\_eq) \\
& \quad c\_2Erealax\_2Ereal\_ABS)\ c\_2Erealax\_2Ereal\_REP))
\end{aligned} \tag{42}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0h \in ty\_2Ehreal\_2Ehreal. (\forall V1i \in ty\_2Ehreal\_2Ehreal. \\
& ((p\ (ap\ (ap\ c\_2Ehreal\_2Ehreal\_lt\ V0h)\ V1i)) \Leftrightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt \\
& \quad (ap\ c\_2Erealax\_2Ereal\_of\_hreal\ V0h))\ (ap\ c\_2Erealax\_2Ereal\_of\_hreal \\
& \quad \quad V1i))))))
\end{aligned}$$