

thm_2Erealax_2EREAL_LT_MUL (TM-
RDW44fZpGMzK74bbhx569KF7A7iV8uE7G)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$
of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ecombin_2E_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 8 We define $c_2Ecombin_2E_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 9 We define $c_2Ecombin_2E_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2E_2ES A_27a (A_27a^{A_27a}) A_27a$

Definition 10 We define $c_2Equotient_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f \in$

Definition 11 We define $c_2Equotient_2E_3D_3D_3D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R1 \in ((2^{A_27a})^{A_27b})$

Definition 12 We define $c_2Ecombin_2E_2EW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in ((A_27b^{A_27a})^{A_27a}).(\lambda V1x \in$

Definition 13 We define $c_2Equotient_2E_2Respects$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(c_2Ecombin_2E_2EW A_27a A_27a$

Definition 14 We define $c_2Ebool_2E_2EIN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Definition 15 We define $c_2Ebool_2E_2ERES_FORALL$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).$

Definition 16 We define $c_2Equotient_2E_2EEQUIV$ to be $\lambda A_27a : \iota.\lambda V0E \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_2EIN$

Definition 17 We define $c_2Ebool_2E_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E_21 2) (\lambda V2t \in$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (2)$$

Let $c_2Erealax_2Etreax_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (3)$$

Definition 18 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (4)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 19 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (7)$$

Definition 20 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$

Definition 21 We define $c_2Ehrat_2Etrat_1$ to be $(ap\ (ap\ (c_2Epair_2E2C\ ty_2Enum_2Enum\ ty_2Enum_2Enum)\ ty_2Enum_2Enum)\ ty_2Enum_2Enum)$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)) \quad (8)$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehrat_2Ehrat \quad (9)$$

Let $c_2Ehrat_2Ehrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_ABS_CLASS \in (ty_2Ehrat_2Ehrat^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})}) \quad (10)$$

Definition 22 We define $c_2Ehrat_2Ehrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2$

Definition 23 We define $c_2Ehrat_2Ehrat_1$ to be $(ap\ c_2Ehrat_2Ehrat_ABS\ c_2Ehrat_2Etrat_1)$.

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \quad (11)$$

Definition 24 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge \dots))$ of type $\iota \Rightarrow \iota$.

Definition 25 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap\ (c_2Emin_2E_40\ (ty_2$

Let $c_2Ehrat_2Etrat_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \quad (12)$$

Definition 26 We define $c_2Ehrat_2Ehrat_add$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat.\lambda V1T2 \in ty_2Ehrat_2E$

Definition 27 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ ($

Definition 28 We define $c_2Ehreal_2Ehrat_lt$ to be $\lambda V0x \in ty_2Ehrat_2Ehrat.\lambda V1y \in ty_2Ehrat_2Ehrat$

Definition 29 We define $c_2Ehreal_2Ecut_of_hrat$ to be $\lambda V0x \in ty_2Ehrat_2Ehrat.(\lambda V1y \in ty_2Ehrat_2E$

Let $c_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$c_2Ehreal_2Ehreal \in (ty_2Ehreal_2Ehreal)^{(2^{ty_2Ehrat_2Ehrat})} \quad (13)$$

Definition 30 We define $c_2Ehreal_2Ehreal_1$ to be $(ap\ c_2Ehreal_2Ehreal\ (ap\ c_2Ehreal_2Ecut_of_hrat\ ($

Definition 31 We define $c_2Erealax_2Etreax_0$ to be $(ap\ (ap\ (c_2Epair_2E_2C\ ty_2Ehreal_2Ehreal\ ty_2Eh$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (14)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (15)$$

Definition 32 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2$

Definition 33 We define $c_2Erealax_2Ereal_0$ to be $(ap\ c_2Erealax_2Ereal_ABS\ c_2Erealax_2Etreax_0)$.

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal))_{ty_2Erealax_2Ereal})_{ty_2Erealax_2Ereal} \quad (16)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}_{ty_2Erealax_2Ereal})_{ty_2Erealax_2Ereal})_{ty_2Erealax_2Ereal} \quad (17)$$

Definition 34 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ ty_2Erealax_2Ereal\ a))$

Definition 35 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(c_2Erealax_2Ereal_mul\ T1\ T2)$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}_{ty_2Erealax_2Ereal})_{ty_2Erealax_2Ereal})_{ty_2Erealax_2Ereal} \quad (18)$$

Definition 36 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(c_2Erealax_2Ereal_lt\ T1\ T2)$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & A_27a\ A_27a)\ (c_2Emin_2E_3D\ A_27a))\ (c_2Ecombin_2EI\ A_27a))\ (\\ & c_2Ecombin_2EI\ A_27a))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& ((\forall V1x \in A_27a. (\forall V2y \in A_27a. ((p (ap (ap V0R V1x) V2y)) \Leftrightarrow \\
& ((ap V0R V1x) = (ap V0R V2y)))))) \Leftrightarrow ((\forall V3x \in A_27a. (p (ap (ap V0R \\
& V3x) V3x))) \wedge ((\forall V4x \in A_27a. (\forall V5y \in A_27a. ((p (ap (\\
& ap V0R V4x) V5y)) \Rightarrow (p (ap (ap V0R V5y) V4x)))))) \wedge (\forall V6x \in A_27a. \\
& (\forall V7y \in A_27a. (\forall V8z \in A_27a. (((p (ap (ap V0R V6x) V7y)) \wedge \\
& (p (ap (ap V0R V7y) V8z))) \Rightarrow (p (ap (ap V0R V6x) V8z))))))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& (2^{A_27a})^{A_27a}). (\forall V1abs1 \in (A_27c^{A_27a}). (\forall V2rep1 \in \\
& (A_27a^{A_27c}). ((p (ap (ap (ap (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& V0R1) V1abs1) V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}). (\forall V4abs2 \in \\
& (A_27d^{A_27b}). (\forall V5rep2 \in (A_27b^{A_27d}). ((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& A_27b\ A_27d) V3R2) V4abs2) V5rep2)) \Rightarrow (p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& (A_27b^{A_27a}) (A_27d^{A_27c})) (ap (ap (c_2Equotient_2E_3D_3D_3D_3E \\
& A_27a\ A_27b) V0R1) V3R2)) (ap (ap (c_2Equotient_2E_2D_2D_3E\ A_27c \\
& A_27b\ A_27a\ A_27d) V2rep1) V4abs2)) (ap (ap (c_2Equotient_2E_2D_2D_3E \\
& A_27a\ A_27d\ A_27c\ A_27b) V1abs1) V5rep2))))))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& (2^{A_27a})^{A_27a}). (\forall V1abs1 \in (A_27c^{A_27a}). (\forall V2rep1 \in \\
& (A_27a^{A_27c}). ((p (ap (ap (ap (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& V0R1) V1abs1) V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}). (\forall V4abs2 \in \\
& (A_27d^{A_27b}). (\forall V5rep2 \in (A_27b^{A_27d}). ((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& A_27b\ A_27d) V3R2) V4abs2) V5rep2)) \Rightarrow (\forall V6f \in (A_27d^{A_27c}). \\
& ((\lambda V7x \in A_27c. (ap V6f V7x)) = (ap (ap (ap (c_2Equotient_2E_2D_2D_3E \\
& A_27c\ A_27b\ A_27a\ A_27d) V2rep1) V4abs2) (\lambda V8x \in A_27a. (ap V5rep2 \\
& (ap V6f (ap V1abs1 V8x))))))))))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0REL \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}). ((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& A_27a\ A_27b) V0REL) V1abs) V2rep)) \Rightarrow (\forall V3x1 \in A_27a. (\forall V4x2 \in \\
& A_27a. ((p (ap (ap V0REL V3x1) V4x2)) \Rightarrow (p (ap (ap V0REL V3x1) (ap V2rep \\
& (ap V1abs V4x2))))))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27b}).((p\ (\\
& ap\ (c_2Ebool_2E_21\ A_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b\ 2)\ V1abs)\ (c_2Ecombin_2EI \\
& \quad 2))\ V3f))))))))) \\
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27a}).(\forall V4g \in \\
& \quad (2^{A_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad 2)\ V0R)\ (c_2Emin_2E_3D\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (\\
& ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0R))\ V4g))))))))) \\
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b^{A_27a}). \\
& \quad (\forall V7g \in (A_27b^{A_27a}).(\forall V8x \in A_27a.(\forall V9y \in \\
& \quad A_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))) \\
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0E \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1P \in (2^{A_27a}).((p\ (ap\ (c_2Equotient_2EEQUIV\ A_27a) \\
& V0E)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E_21\ A_27a)\ V1P)))))) \\
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)). \\
& (\forall V1q \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)). \\
& ((p\ (ap\ (ap\ c_2Erealax_2Etreal_eq\ V0p)\ V1q)) \Leftrightarrow ((ap\ c_2Erealax_2Etreal_eq \\
& \quad V0p) = (ap\ c_2Erealax_2Etreal_eq\ V1q))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)). \\
& (\forall V1y \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)). \\
& (((p\ (ap\ (ap\ c_2Erealax_2Etreal_lt\ c_2Erealax_2Etreal_0)\ V0x)) \wedge \\
& \quad (p\ (ap\ (ap\ c_2Erealax_2Etreal_lt\ c_2Erealax_2Etreal_0)\ V1y))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ c_2Erealax_2Etreal_lt\ c_2Erealax_2Etreal_0)\ (ap \\
& \quad \quad (ap\ c_2Erealax_2Etreal_mul\ V0x)\ V1y))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)). \\
& (\forall V1x2 \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)). \\
& (\forall V2y1 \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)). \\
& (\forall V3y2 \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)). \\
& (((p\ (ap\ (ap\ c_2Erealax_2Etreal_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c_2Erealax_2Etreal_eq \\
& \quad V2y1)\ V3y2))) \Rightarrow (p\ (ap\ (ap\ c_2Erealax_2Etreal_eq\ (ap\ (ap\ c_2Erealax_2Etreal_mul \\
& \quad \quad V0x1)\ V2y1))\ (ap\ (ap\ c_2Erealax_2Etreal_mul\ V1x2)\ V3y2))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)). \\
& (\forall V1x2 \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)). \\
& (\forall V2y1 \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)). \\
& (\forall V3y2 \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)). \\
& (((p\ (ap\ (ap\ c_2Erealax_2Etreal_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c_2Erealax_2Etreal_eq \\
& \quad V2y1)\ V3y2))) \Rightarrow ((p\ (ap\ (ap\ c_2Erealax_2Etreal_lt\ V0x1)\ V2y1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ c_2Erealax_2Etreal_lt\ V1x2)\ V3y2))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\
& \quad ty_2Ehreal_2Ehreal)\ ty_2Erealax_2Ereal)\ c_2Erealax_2Etreal_eq) \\
& \quad \quad c_2Erealax_2Ereal_ABS)\ c_2Erealax_2Ereal_REP))
\end{aligned} \tag{37}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ c_2Erealax_2Ereal_0)\ V0x)) \wedge \\
& \quad (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ c_2Erealax_2Ereal_0)\ V1y))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ c_2Erealax_2Ereal_0)\ (ap\ (ap \\
& \quad \quad c_2Erealax_2Ereal_mul\ V0x)\ V1y))))))
\end{aligned}$$