

thm\_2Erealax\_2EREAL\_\_MUL\_\_LID  
(TMYZHLTzBmrZ3UfqQvaA6d2ksvKLwLCmW7v)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$   
of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 7** We define  $c\_2Ecombin\_2E\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 8** We define  $c\_2Ecombin\_2E\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 9** We define  $c\_2Ecombin\_2E\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2E\_2ES A\_27a (A\_27a^{A\_27a}) A\_27a$

**Definition 10** We define  $c\_2Equotient\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f \in$

**Definition 11** We define  $c\_2Equotient\_2E\_3D\_3D\_3D\_3E$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27b})^{A\_27a}$

**Definition 12** We define  $c\_2Ecombin\_2E\_2EW$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x \in$

**Definition 13** We define  $c\_2Equotient\_2E\_2Respects$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(c\_2Ecombin\_2E\_2EW A\_27a A\_27a$

**Definition 14** We define  $c\_2Ebool\_2E\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x))$

**Definition 15** We define  $c\_2Ebool\_2E\_2ERES\_FORALL$  to be  $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).$

**Definition 16** We define  $c\_2Equotient\_2E\_2EEQUIV$  to be  $\lambda A\_27a : \iota.\lambda V0E \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2E\_2EIN$

**Definition 17** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$



**Definition 22** We define  $c\_2Ehrat\_2Ehrat\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

**Definition 23** We define  $c\_2Ehrat\_2Ehrat\_1$  to be  $(ap\ c\_2Ehrat\_2Ehrat\_ABS\ c\_2Ehrat\_2Etrat\_1)$ .

Let  $c\_2Ehrat\_2Ehrat\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehrat}) \quad (11)$$

**Definition 24** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p\ (ap\ P\ x))$  **then**  $(the\ (\lambda x.x \in A \wedge p\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 25** We define  $c\_2Ehrat\_2Ehrat\_REP$  to be  $\lambda V0a \in ty\_2Ehrat\_2Ehrat.(ap\ (c\_2Emin\_2E\_40\ (ty\_2$

Let  $c\_2Ehrat\_2Etrat\_add : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum})^{ty\_2Ehrat\_2Ehrat}) \quad (12)$$

**Definition 26** We define  $c\_2Ehrat\_2Ehrat\_add$  to be  $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2Ehrat$

**Definition 27** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ ($

**Definition 28** We define  $c\_2Ehreal\_2Ehreal\_lt$  to be  $\lambda V0x \in ty\_2Ehrat\_2Ehrat.\lambda V1y \in ty\_2Ehrat\_2Ehrat$

**Definition 29** We define  $c\_2Ehreal\_2Ecut\_of\_hreal$  to be  $\lambda V0x \in ty\_2Ehrat\_2Ehrat.(\lambda V1y \in ty\_2Ehrat\_2Ehrat$

Let  $c\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$c\_2Ehreal\_2Ehreal \in (ty\_2Ehreal\_2Ehreal^{(2^{ty\_2Ehreal\_2Ehreal})}) \quad (13)$$

**Definition 30** We define  $c\_2Ehreal\_2Ehreal\_1$  to be  $(ap\ c\_2Ehreal\_2Ehreal\ (ap\ c\_2Ehreal\_2Ecut\_of\_hreal\ ($

Let  $c\_2Ehreal\_2Ecut : \iota$  be given. Assume the following.

$$c\_2Ehreal\_2Ecut \in ((2^{ty\_2Ehreal\_2Ehreal})^{ty\_2Ehreal\_2Ehreal}) \quad (14)$$

**Definition 31** We define  $c\_2Ehreal\_2Ehreal\_add$  to be  $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2Ehreal$

**Definition 32** We define  $c\_2Erealax\_2Etreax\_1$  to be  $(ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal$

Let  $ty\_2Erealax\_2Etreax : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Etreax \quad (15)$$

Let  $c\_2Erealax\_2Etreax\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreax\_ABS\_CLASS \in (ty\_2Erealax\_2Etreax^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})}) \quad (16)$$

**Definition 33** We define  $c\_2Erealax\_2Etreax\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2$

**Definition 34** We define  $c\_2Erealax\_2Ereal\_1$  to be  $(ap\ c\_2Erealax\_2Ereal\_ABS\ c\_2Erealax\_2Ereal\_1)$ .

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (17)$$

**Definition 35** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ ($

Let  $c\_2Erealax\_2Ereal\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Erealax})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)} \quad (18)$$

**Definition 36** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (c\_2Ecombin\_2EI\ A\_27a)\ V0x) = V0x)) \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ & A\_27a\ A\_27a)\ (c\_2Emin\_2E3D\ A\_27a))\ (c\_2Ecombin\_2EI\ A\_27a))\ ( \\ & c\_2Ecombin\_2EI\ A\_27a))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& ((\forall V1x \in A\_27a. (\forall V2y \in A\_27a. ((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Leftrightarrow \\
& ((ap\ V0R\ V1x) = (ap\ V0R\ V2y)))))) \Leftrightarrow ((\forall V3x \in A\_27a. (p\ (ap\ (ap\ V0R \\
& V3x)\ V3x))) \wedge ((\forall V4x \in A\_27a. (\forall V5y \in A\_27a. ((p\ (ap\ ( \\
& ap\ V0R\ V4x)\ V5y)) \Rightarrow (p\ (ap\ (ap\ V0R\ V5y)\ V4x)))))) \wedge (\forall V6x \in A\_27a. \\
& (\forall V7y \in A\_27a. (\forall V8z \in A\_27a. (((p\ (ap\ (ap\ V0R\ V6x)\ V7y)) \wedge \\
& (p\ (ap\ (ap\ V0R\ V7y)\ V8z))) \Rightarrow (p\ (ap\ (ap\ V0R\ V6x)\ V8z))))))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\
& (2^{A\_27a})^{A\_27a}). (\forall V1abs1 \in (A\_27c^{A\_27a}). (\forall V2rep1 \in \\
& (A\_27a^{A\_27c}). ((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\
& V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}). (\forall V4abs2 \in \\
& (A\_27d^{A\_27b}). (\forall V5rep2 \in (A\_27b^{A\_27d}). ((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& (A\_27b^{A\_27a})\ (A\_27d^{A\_27c}))\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E \\
& A\_27a\ A\_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E\ A\_27c \\
& A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E \\
& A\_27a\ A\_27d\ A\_27c\ A\_27b)\ V1abs1)\ V5rep2))))))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}). ((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x \in A\_27b. (\forall V4y \in \\
& A\_27b. ((V3x = V4y) \Leftrightarrow (p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3x))\ (ap\ V2rep\ V4y))))))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}). ((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A\_27a. (\forall V4x2 \in \\
& A\_27a. (\forall V5y1 \in A\_27a. (\forall V6y2 \in A\_27a. (((p\ (ap\ (ap\ V0R \\
& V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow \\
& (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2))))))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\
& \quad (2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\
& \quad (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in \\
& \quad (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27d^{A\_27c}). \\
& \quad ((\lambda V7x \in A\_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E \\
& \quad A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A\_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x))))))))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0REL \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& \quad (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A\_27a.(\forall V4x2 \in \\
& \quad A\_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2))))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& \quad (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27b}).((p\ ( \\
& \quad ap\ (c\_2Ebool\_2E\_21\ A\_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL \\
& \quad A\_27a)\ (ap\ (c\_2Equotient\_2Erespects\ A\_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c\_2Equotient\_2E\_2D\_2D\_3E\ A\_27a\ 2\ A\_27b\ 2)\ V1abs)\ (c\_2Ecombin\_2EI \\
& \quad 2))\ V3f))))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& \quad (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27a}).(\forall V4g \in \\
& \quad (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E\ A\_27a \\
& \quad 2)\ V0R)\ (c\_2Emin\_2E\_3D\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL \\
& \quad A\_27a)\ (ap\ (c\_2Equotient\_2Erespects\ A\_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ ( \\
& \quad ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL\ A\_27a)\ (ap\ (c\_2Equotient\_2Erespects \\
& \quad A\_27a\ 2)\ V0R))\ V4g))))))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\
& \quad (2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\
& \quad (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in \\
& \quad (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27b^{A\_27a}). \\
& \quad (\forall V7g \in (A\_27b^{A\_27a}).(\forall V8x \in A\_27a.(\forall V9y \in \\
& \quad A\_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E\ A\_27a \\
& \quad A\_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ ( \\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0E \in ((2^{A\_27a})^{A\_27a}). \\
& \quad (\forall V1P \in (2^{A\_27a}).((p\ (ap\ (c\_2Equotient\_2EEQUIV\ A\_27a) \\
& \quad V0E)) \Rightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL\ A\_27a)\ (ap\ (c\_2Equotient\_2Erespects \\
& \quad A\_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c\_2Ebool\_2E.21\ A\_27a)\ V1P))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad (\forall V1q \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad ((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_eq\ V0p)\ V1q)) \Leftrightarrow ((ap\ c\_2Erealax\_2Etreal\_eq \\
& \quad V0p) = (ap\ c\_2Erealax\_2Etreal\_eq\ V1q))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_eq\ (ap\ (ap\ c\_2Erealax\_2Etreal\_mul \\
& \quad c\_2Erealax\_2Etreal\_1)\ V0x))\ V0x)))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad (\forall V1x2 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad (\forall V2y1 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad (\forall V3y2 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad (((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_eq \\
& \quad V2y1)\ V3y2))) \Rightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_eq\ (ap\ (ap\ c\_2Erealax\_2Etreal\_mul \\
& \quad V0x1)\ V2y1))\ (ap\ (ap\ c\_2Erealax\_2Etreal\_mul\ V1x2)\ V3y2))))))
\end{aligned} \tag{37}$$

Assume the following.

$$(p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) ty\_2Erealax\_2Ereal) c\_2Erealax\_2Etrealm\_eq) c\_2Erealax\_2Ereal\_ABS) c\_2Erealax\_2Ereal\_REP)) \quad (38)$$

**Theorem 1**

$$(\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul c\_2Erealax\_2Ereal\_1) V0x) = V0x))$$