thm_2Erealax_2EREAL__SUP__ALLPOS (TMFndGGziwTHjGT-guMkmnA9SezyNTwyD1Q5)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Ehrat\_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehrat\_2Ehrat \tag{1}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{2}$$

Let $c\_2Ehreal\_2Ecut : \iota$ be given. Assume the following.

$$c\_2Ehreal\_2Ecut \in ((2^{ty\_2Ehrat\_2Ehrat})^{ty\_2Ehreal\_2Ehreal}) \tag{3}$$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 7 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A$

Let $c\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$c\_2Ehreal\_2Ehreal \in (ty\_2Ehreal\_2Ehreal^{(2^{ty\_2Ehrat\_2Ehrat})}) \tag{4}$$

Definition 8 *We define* c_2Ehreal_2Ehreal__sup *to be* $\lambda V0P \in (2^{ty\_2Ehreal\_2Ehreal}).(ap\ c\_2Ehreal\_2Ehreal$

Definition 9 *We define* $\mathsf{c\_2Ecombin\_2EK}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 10 *We define* $\mathsf{c\_2Ecombin\_2ES}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 11 *We define* $\mathsf{c\_2Ecombin\_2EI}$ *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 12 *We define* $\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f$

Definition 13 *We define* $\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_2}$

Definition 14 *We define* $\mathsf{c\_2Ecombin\_2EW}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x$

Definition 15 *We define* $\mathsf{c\_2Equotient\_2Erespects}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\mathsf{c\_2Ecombin\_2EW}\ A\_27a\ A\_2$

Definition 16 *We define* $\mathsf{c\_2Ebool\_2EIN}$ *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 17 *We define* $\mathsf{c\_2Ebool\_2ERES\_\_FORALL}$ *to be* $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).($

Definition 18 *We define* $\mathsf{c\_2Equotient\_2EEQUIV}$ *to be* $\lambda A\_27a : \iota.\lambda V0E \in ((2^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E}$

Let $ty\_2Epair\_2Eprod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (5)$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \quad (6)$$

Definition 19 *We define* $\mathsf{c\_2Equotient\_2EQUOTIENT}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (7)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (8)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (9)$$

Definition 20 *We define* $\mathsf{c\_2Enum\_2E0}$ *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Epair\_2EABS\_\_prod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (10)$$

Definition 21 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 22 *We define* c_2Ehrat_2Etrat__1 *to be* $(ap\ (ap\ ($c_2Epair_2E_2C $ty\_2Enum\_2Enum\ ty\_2Enum\_$

Let $c\_2Ehrat\_2Etrat\_\_eq : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \quad (11)$$

Let $c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS \in (ty\_2Ehrat\_2Ehrat^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}) \quad (12)$$

Definition 23 *We define* c_2Ehrat_2Ehrat__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 24 *We define* c_2Ehrat_2Ehrat__1 *to be* $(ap$ c_2Ehrat_2Ehrat__ABS c_2Ehrat_2Etrat__1$)$.

Let $c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehra} \quad (13)$$

Definition 25 *We define* c_2Ehrat_2Ehrat__REP *to be* $\lambda V0a \in ty\_2Ehrat\_2Ehrat.(ap$ (c_2Emin_2E_40 $(ty\_2$

Let $c\_2Ehrat\_2Etrat\_\_add : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty} \quad (14)$$

Definition 26 *We define* c_2Ehrat_2Ehrat__add *to be* $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2E$

Definition 27 *We define* c_2Ehreal_2Ehrat__lt *to be* $\lambda V0x \in ty\_2Ehrat\_2Ehrat.\lambda V1y \in ty\_2Ehrat\_2Ehrat$

Definition 28 *We define* c_2Ehreal_2Ecut__of__hrat *to be* $\lambda V0x \in ty\_2Ehrat\_2Ehrat.(\lambda V1y \in ty\_2Ehrat\_2$

Definition 29 *We define* c_2Ehreal_2Ehreal__1 *to be* $(ap\ c\_2Ehreal\_2Ehreal\ (ap$ c_2Ehreal_2Ecut__of__hrat

Definition 30 *We define* c_2Erealax_2Etreal__0 *to be* $(ap\ (ap$ (c_2Epair_2E_2C $ty\_2Ehreal\_2Ehreal\ ty\_2Eh$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \quad (15)$$

Definition 31 *We define* c_2Ehreal_2Ehreal__add *to be* $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_$

Definition 32 *We define* c_2Erealax_2Etreal__of__hreal *to be* $\lambda V0x \in ty\_2Ehreal\_2Ehreal.(ap\ (ap$ (c_2Epair

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{16}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{17}$$

Definition 33 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{18}$$

Definition 34 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Ehreal\_\_of\_\_treal : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ehreal\_\_of\_\_treal \in (ty\_2Ehreal\_2Ehreal^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehre} \tag{19}$$

Definition 35 *We define* c_2Erealax_2Ehreal__of__real *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_$

Definition 36 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 37 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 38 *We define* c_2Ehreal_2Ehreal__lt *to be* $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2E$

Definition 39 *We define* c_2Erealax_2Ereal__of__hreal *to be* $\lambda V0T1 \in ty\_2Ehreal\_2Ehreal.(ap$ c_2Erealax_2

Definition 40 *We define* c_2Erealax_2Ereal__0 *to be* $(ap$ c_2Erealax_2Ereal__ABS c_2Erealax_2Etreal__0$)$.

Definition 41 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 42 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Assume the following.

$$True \tag{20}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow(p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge(((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{27}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{29}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Ehreal\_2Ehreal}).(((\exists V1X \in ty\_2Ehreal\_2Ehreal.(p\ (ap\ V0P\ V1X)))\wedge(\exists V2Y \in ty\_2Ehreal\_2Ehreal.(\forall V3X \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ V0P\ V3X))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V3X)\ V2Y))))))\Rightarrow(\forall V4Y \in ty\_2Ehreal\_2Ehreal.((\exists V5X \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ V0P\ V5X))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V4Y)\ V5X))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V4Y)\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_sup}\ V0P)))))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27a)\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a))\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a))\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).((\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Leftrightarrow ((ap\ V0R\ V1x) = (ap\ V0R\ V2y)))))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ (ap\ V0R\ V3x)\ V3x))) \wedge ((\forall V4x \in A\_27a.(\forall V5y \in A\_27a.((p\ (ap\ (ap\ V0R\ V4x)\ V5y)) \Rightarrow (p\ (ap\ (ap\ V0R\ V5y)\ V4x)))))) \wedge (\forall V6x \in A\_27a.(\forall V7y \in A\_27a.(\forall V8z \in A\_27a.(((p\ (ap\ (ap\ V0R\ V6x)\ V7y)) \wedge (p\ (ap\ (ap\ V0R\ V7y)\ V8z))) \Rightarrow (p\ (ap\ (ap\ V0R\ V6x)\ V8z)))))))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27c)\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ (A\_27b^{A\_27a})\ (A\_27d^{A\_27c}))\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ A\_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27a\ A\_27d\ A\_27c\ A\_27b)\ V1abs1)\ V5rep2)))))))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}).(\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x \in A\_27b.(\forall V4y \in A\_27b.((V3x = V4y) \Leftrightarrow (p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3x))\ (ap\ V2rep\ V4y)))))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}).(\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A\_27a.(\forall V4x2 \in A\_27a.(\forall V5y1 \in A\_27a.(\forall V6y2 \in A\_27a.(((p\ (ap\ (ap\ V0R\ V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2)))))))))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27c)\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27d^{A\_27c}).\ ((\lambda V7x \in A\_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A\_27a.(ap\ V5rep2\ (ap\ V6f\ (ap\ V1abs1\ V8x)))))))))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\ \forall V0REL \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}).\ (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A\_27a.(\forall V4x2 \in A\_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep\ (ap\ V1abs\ V4x2))))))))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\ \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}).\ (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27b}).((p\ (\ ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27a\ 2\ A\_27b\ 2)\ V1abs)\ (\mathsf{c\_2Ecombin\_2EI}\ 2))\ V3f))))))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\ \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}).\ (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27a}).(\forall V4g \in (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ 2)\ V0R)\ (\mathsf{c\_2Emin\_2E\_3D}\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (\ ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ V4g)))))))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27c)\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27b^{A\_27a}). (\forall V7g \in (A\_27b^{A\_27a}).(\forall V8x \in A\_27a.(\forall V9y \in A\_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ A\_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \land (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))))))))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0E \in ((2^{A\_27a})^{A\_27a}). (\forall V1P \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Equotient\_2EEQUIV}\ A\_27a)\ V0E)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27a)\ V1P)))))) \tag{41}$$

Assume the following.

$$(\forall V0p \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). (\forall V1q \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). ((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_\_eq\ V0p)\ V1q)) \Leftrightarrow ((ap\ c\_2Erealax\_2Etreal\_\_eq\ V0p) = (ap\ c\_2Erealax\_2Etreal\_\_eq\ V1q))))) \tag{42}$$

Assume the following.

$$((\forall V0h \in ty\_2Ehreal\_2Ehreal.((ap\ c\_2Erealax\_2Ehreal\_\_of\_\_treal\ (ap\ \mathsf{c\_2Erealax\_2Etreal\_\_of\_\_hreal}\ V0h)) = V0h)) \land (\forall V1r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). ((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_\_lt\ \mathsf{c\_2Erealax\_2Etreal\_\_0})\ V1r)) \Leftrightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_\_eq\ (ap\ \mathsf{c\_2Erealax\_2Etreal\_\_of\_\_hreal}\ (ap\ c\_2Erealax\_2Ehreal\_\_of\_\_treal\ V1r)))\ V1r))))) \tag{43}$$

Assume the following.

$$(\forall V0h \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). (\forall V1i \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). ((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_\_eq\ V0h)\ V1i)) \Rightarrow ((ap\ c\_2Erealax\_2Ehreal\_\_of\_\_treal\ V0h) = (ap\ c\_2Erealax\_2Ehreal\_\_of\_\_treal\ V1i))))) \tag{44}$$

Assume the following.

$$(\forall V0x1 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). (\forall V1x2 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). (\forall V2y1 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). (\forall V3y2 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). (((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_\_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_\_eq\ V2y1)\ V3y2))) \Rightarrow ((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_\_lt\ V0x1)\ V2y1)) \Leftrightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_\_lt\ V1x2)\ V3y2)))))))) \tag{45}$$

Assume the following.

$$(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)\ ty\_2Erealax\_2Ereal)\ c\_2Erealax\_2Etreal\_\_eq)\ \mathsf{c\_2Erealax\_2Ereal\_\_ABS})\ \mathsf{c\_2Erealax\_2Ereal\_\_REP})) \tag{46}$$

Assume the following.

$$(\forall V0h \in ty\_2Ehreal\_2Ehreal.(\forall V1i \in ty\_2Ehreal\_2Ehreal. ((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V0h)\ V1i)) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_of\_\_hreal}\ V0h))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_of\_\_hreal}\ V1i)))))) \tag{47}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1y \in ty\_2Erealax\_2Ereal. ((\forall V2x \in ty\_2Erealax\_2Ereal.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ \mathsf{c\_2Erealax\_2Ereal\_\_0})\ V2x)))) \Rightarrow ((\exists V3x \in ty\_2Erealax\_2Ereal.((p\ (ap\ V0P\ V3x)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1y)\ V3x)))) \Leftrightarrow (\exists V4X \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ V0P\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_of\_\_hreal}\ V4X))) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1y)\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_of\_\_hreal}\ V4X))))))))) \tag{48}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1X \in ty\_2Ehreal\_2Ehreal. ((p\ (ap\ V0P\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_of\_\_hreal}\ V1X))) \Leftrightarrow (p\ (ap\ (\lambda V2h \in ty\_2Ehreal\_2Ehreal.(ap\ V0P\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_of\_\_hreal}\ V2h)))\ V1X))))) \tag{49}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Erealax\_2Ereal}).(((\forall V1x \in ty\_2Erealax\_2Ereal.((p\ (ap\ V0P\ V1x))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ \mathsf{c\_2Erealax\_2Ereal\_\_0})\ V1x))))\wedge((\exists V2x \in ty\_2Erealax\_2Ereal.(p\ (ap\ V0P\ V2x)))\wedge(\exists V3z \in ty\_2Erealax\_2Ereal.(\forall V4x \in ty\_2Erealax\_2Ereal.((p\ (ap\ V0P\ V4x))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V4x)\ V3z)))))))\Rightarrow((\exists V5X \in ty\_2Ehreal\_2Ehreal.(p\ (ap\ (\lambda V6h \in ty\_2Ehreal\_2Ehreal.(ap\ V0P\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_of\_\_hreal}\ V6h)))\ V5X)))\wedge(\exists V7Y \in ty\_2Ehreal\_2Ehreal.(\forall V8X \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ (\lambda V9h \in ty\_2Ehreal\_2Ehreal.(ap\ V0P\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_of\_\_hreal}\ V9h)))\ V8X))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V8X)\ V7Y)))))))) \tag{50}$$

Assume the following.

$$(\forall V0y \in ty\_2Erealax\_2Ereal.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ \mathsf{c\_2Erealax\_2Ereal\_\_0})\ V0y)))\Rightarrow(\forall V1x \in ty\_2Ehreal\_2Ehreal.(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0y)\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_of\_\_hreal}\ V1x)))))) \tag{51}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{52}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{53}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow(((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{54}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{55}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{60}$$

Theorem 1

$$(\forall V0P \in (2^{ty\_2Erealax\_2Ereal}).(((\forall V1x \in ty\_2Erealax\_2Ereal. ((p\ (ap\ V0P\ V1x))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ \mathsf{c\_2Erealax\_2Ereal\_\_0})\ V1x))))\wedge((\exists V2x \in ty\_2Erealax\_2Ereal.(p\ (ap\ V0P\ V2x)))\wedge (\exists V3z \in ty\_2Erealax\_2Ereal.(\forall V4x \in ty\_2Erealax\_2Ereal. ((p\ (ap\ V0P\ V4x))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V4x)\ V3z)))))))\Rightarrow (\exists V5s \in ty\_2Erealax\_2Ereal.(\forall V6y \in ty\_2Erealax\_2Ereal. ((\exists V7x \in ty\_2Erealax\_2Ereal.((p\ (ap\ V0P\ V7x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V6y)\ V7x))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V6y)\ V5s)))))))$$