

thm_2Erealax_2ESUP__ALLPOS__LEMMA2 (TMS3tt4FM2edtcqf3tZmxb9c8uXPX4q6Nv7)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1Q \in 2.V1Q))\ (\lambda V2t \in 2.V2t))\ (\lambda V3t \in 2.V3t))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V2t \in 2.V2t))\ (\lambda V3t \in 2.V3t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{4}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{5}$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2E$

Definition 8 We define $c_2Ehrat_2Etrat_1$ to be $(ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (6)$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehrat_2Ehrat \quad (7)$$

Let $c_2Ehrat_2Ehrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_ABS_CLASS \in (ty_2Ehrat_2Ehrat^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})}) \quad (8)$$

Definition 9 We define $c_2Ehrat_2Ehrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2E$

Definition 10 We define $c_2Ehrat_2Ehrat_1$ to be $(ap\ c_2Ehrat_2Ehrat_ABS\ c_2Ehrat_2Etrat_1)$.

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \quad (9)$$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap (c_2Emin_2E_40 (ty_2$

Let $c_2Ehrat_2Etrat_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_add \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{ty_2Epair_2Eprod ty_2Enum_2Enum})^{ty_2Epair_2Eprod ty_2Enum_2Enum}) \quad (10)$$

Definition 13 We define $c_2Ehrat_2Ehrat_add$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat.\lambda V1T2 \in ty_2Ehrat_2E$

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P (ap (c_2Emin_2E_40$

Definition 15 We define $c_2Ehreal_2Ehrat_lt$ to be $\lambda V0x \in ty_2Ehrat_2Ehrat.\lambda V1y \in ty_2Ehrat_2Ehrat$

Definition 16 We define $c_2Ehreal_2Ecut_of_hrat$ to be $\lambda V0x \in ty_2Ehrat_2Ehrat.(\lambda V1y \in ty_2Ehrat_2$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (11)$$

Let $c_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$c_2Ehreal_2Ehreal \in (ty_2Ehreal_2Ehreal^{(2^{ty_2Ehrat_2Ehrat})}) \quad (12)$$

Definition 17 We define $c_2Ehreal_2Ehreal_1$ to be $(ap\ c_2Ehreal_2Ehreal\ (ap\ c_2Ehreal_2Ecut_of_hreal\ \iota))$.
Let $c_2Ehreal_2Ecut : \iota$ be given. Assume the following.

$$c_2Ehreal_2Ecut \in ((2^{ty_2Ehreal_2Ehreal})^{ty_2Ehreal_2Ehreal}) \quad (13)$$

Definition 18 We define $c_2Ehreal_2Ehreal_add$ to be $\lambda V0X \in ty_2Ehreal_2Ehreal.\lambda V1Y \in ty_2Ehreal_2Ehreal.$

Definition 19 We define $c_2Erealax_2Etreax_of_hreal$ to be $\lambda V0x \in ty_2Ehreal_2Ehreal.(ap\ (ap\ (c_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))$

Let $c_2Erealax_2Etreax_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (14)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (15)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (16)$$

Definition 20 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 21 We define $c_2Erealax_2Ereal_of_hreal$ to be $\lambda V0T1 \in ty_2Ehreal_2Ehreal.(ap\ c_2Erealax_2Ereal_of_hreal\ \iota)$

Theorem 1

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Erealax_2Ereal}).(\forall V1X \in ty_2Ehreal_2Ehreal. \\ & ((p\ (ap\ V0P\ (ap\ c_2Erealax_2Ereal_of_hreal\ V1X))) \Leftrightarrow (p\ (ap\ (\lambda V2h \in \\ & ty_2Ehreal_2Ehreal.(ap\ V0P\ (ap\ c_2Erealax_2Ereal_of_hreal \\ & V2h)))\ V1X)))))) \end{aligned}$$