

# thm\_2Erealax\_2ESUP\_\_ALLPOS\_\_LEMMA3 (TMEqNAopBcKtVgUNjhq2Vefbf2bcWTjN5kB)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 4** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27$

**Definition 6** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$   
of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 9** We define  $c\_2Ecombin\_2E\_2K$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 10** We define  $c\_2Ecombin\_2E\_2S$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27$

**Definition 11** We define  $c\_2Ecombin\_2E\_2I$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2E\_2S A\_27a (A\_27a^{A\_27a}) A$

**Definition 12** We define  $c\_2Equotient\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f$

**Definition 13** We define  $c\_2Equotient\_2E\_3D\_3D\_3D\_3E$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27$

**Definition 14** We define  $c\_2Ecombin\_2E\_2W$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x$

**Definition 15** We define  $c\_2Equotient\_2E\_2respects$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(c\_2Ecombin\_2E\_2W A\_27a A\_27$

**Definition 16** We define  $c\_2Ebool\_2E\_2IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x))$

**Definition 17** We define  $c\_Ebool\_ERES\_FORALL$  to be  $\lambda A\_27a : \iota. (\lambda V0p \in (2^{A\_27a}). (\lambda V1m \in (2^{A\_27a}).$

**Definition 18** We define  $c\_Equotient\_EEQUIV$  to be  $\lambda A\_27a : \iota. \lambda V0E \in ((2^{A\_27a})^{A\_27a}). (ap (c\_Ebool\_2E$

**Definition 19** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_Ebool\_2E\_21 2) (\lambda V2t \in$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Erealax\_2Etreax\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreax\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \tag{3}$$

**Definition 20** We define  $c\_Equotient\_2EQUOTIENT$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}). \lambda$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{4}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{5}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{6}$$

**Definition 21** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{7}$$

**Definition 22** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2$

**Definition 23** We define  $c\_2Ehrat\_2Etrat\_1$  to be  $(ap (ap (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum$

Let  $c\_2Ehrat\_2Etrat\_eq : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)) \tag{8}$$

Let  $ty\_2Ehtrat\_2Ehtrat : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehtrat\_2Ehtrat \quad (9)$$

Let  $c\_2Ehtrat\_2Ehtrat\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Ehtrat\_2Ehtrat\_ABS\_CLASS \in (ty\_2Ehtrat\_2Ehtrat^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}) \quad (10)$$

**Definition 24** We define  $c\_2Ehtrat\_2Ehtrat\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$

**Definition 25** We define  $c\_2Ehtrat\_2Ehtrat\_1$  to be  $(ap\ c\_2Ehtrat\_2Ehtrat\_ABS\ c\_2Ehtrat\_2Etrat\_1)$ .

Let  $c\_2Ehtrat\_2Ehtrat\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Ehtrat\_2Ehtrat\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Ehtrat\_2Ehtrat}) \quad (11)$$

**Definition 26** We define  $c\_2Ehtrat\_2Ehtrat\_REP$  to be  $\lambda V0a \in ty\_2Ehtrat\_2Ehtrat.(ap\ (c\_2Emin\_2E40\ (ty\_2Enum\_2Enum)))$

Let  $c\_2Ehtrat\_2Etrat\_add : \iota$  be given. Assume the following.

$$c\_2Ehtrat\_2Etrat\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum})^{ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 27** We define  $c\_2Ehtrat\_2Ehtrat\_add$  to be  $\lambda V0T1 \in ty\_2Ehtrat\_2Ehtrat.\lambda V1T2 \in ty\_2Ehtrat\_2Etrat$

**Definition 28** We define  $c\_2Ehreal\_2Ehtrat\_lt$  to be  $\lambda V0x \in ty\_2Ehtrat\_2Ehtrat.\lambda V1y \in ty\_2Ehtrat\_2Ehtrat$

**Definition 29** We define  $c\_2Ehreal\_2Ecut\_of\_htrat$  to be  $\lambda V0x \in ty\_2Ehtrat\_2Ehtrat.(\lambda V1y \in ty\_2Ehtrat\_2Ehtrat)$

Let  $c\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$c\_2Ehreal\_2Ehreal \in (ty\_2Ehreal\_2Ehreal^{(2^{ty\_2Ehtrat\_2Ehtrat})}) \quad (13)$$

**Definition 30** We define  $c\_2Ehreal\_2Ehreal\_1$  to be  $(ap\ c\_2Ehreal\_2Ehreal\ (ap\ c\_2Ehreal\_2Ecut\_of\_htrat))$

**Definition 31** We define  $c\_2Erealax\_2Etreax\_0$  to be  $(ap\ (ap\ (c\_2Epair\_2E2C\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal)))$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (14)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})}) \quad (15)$$

**Definition 32** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 33** We define  $c\_2Erealax\_2Ereal\_0$  to be  $(ap\ c\_2Erealax\_2Ereal\_ABS\ c\_2Erealax\_2Etreax\_0)$ .

Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (16)$$

Let  $c\_2Ehreal\_2Ecut : \iota$  be given. Assume the following.

$$c\_2Ehreal\_2Ecut \in ((2^{ty\_2Ehreal\_2Ehreal})^{ty\_2Ehreal\_2Ehreal}) \quad (17)$$

**Definition 34** We define  $c\_2Ehreal\_2Ehreal\_add$  to be  $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2Ehreal.$

**Definition 35** We define  $c\_2Erealax\_2Etreal\_of\_hreal$  to be  $\lambda V0x \in ty\_2Ehreal\_2Ehreal.(ap\ (ap\ (c\_2Epair$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal\_REP\_CLASS}) \quad (18)$$

**Definition 36** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E.40\ (t$

Let  $c\_2Erealax\_2Ehreal\_of\_treal : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ehreal\_of\_treal \in (ty\_2Ehreal\_2Ehreal^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (19)$$

**Definition 37** We define  $c\_2Erealax\_2Ehreal\_of\_real$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

**Definition 38** We define  $c\_2Erealax\_2Ereal\_of\_hreal$  to be  $\lambda V0T1 \in ty\_2Ehreal\_2Ehreal.(ap\ c\_2Erealax\_2Ereal$

**Definition 39** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 40** We define  $c\_2Ehreal\_2Ehreal\_lt$  to be  $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2Ehreal$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap\ (c\_2Ecombin\_2EI\ A\_27a)\ V0x) = V0x)) \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ & A\_27a\ A\_27a)\ (c\_2Emin\_2E\_3D\ A\_27a))\ (c\_2Ecombin\_2EI\ A\_27a))\ ( \\ & c\_2Ecombin\_2EI\ A\_27a))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\ & ((\forall V1x \in A\_27a. (\forall V2y \in A\_27a. ((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Leftrightarrow \\ & ((ap\ V0R\ V1x) = (ap\ V0R\ V2y)))))) \Leftrightarrow ((\forall V3x \in A\_27a. (p\ (ap\ (ap\ V0R \\ & V3x)\ V3x))) \wedge ((\forall V4x \in A\_27a. (\forall V5y \in A\_27a. ((p\ (ap\ ( \\ & ap\ V0R\ V4x)\ V5y)) \Rightarrow (p\ (ap\ (ap\ V0R\ V5y)\ V4x)))))) \wedge (\forall V6x \in A\_27a. \\ & (\forall V7y \in A\_27a. (\forall V8z \in A\_27a. (((p\ (ap\ (ap\ V0R\ V6x)\ V7y)) \wedge \\ & (p\ (ap\ (ap\ V0R\ V7y)\ V8z))) \Rightarrow (p\ (ap\ (ap\ V0R\ V6x)\ V8z)))))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\ & (2^{A\_27a})^{A\_27a}). (\forall V1abs1 \in (A\_27c^{A\_27a}). (\forall V2rep1 \in \\ & (A\_27a^{A\_27c}). ((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\ & V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}). (\forall V4abs2 \in \\ & (A\_27d^{A\_27b}). (\forall V5rep2 \in (A\_27b^{A\_27d}). ((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ & A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ & (A\_27b^{A\_27a})\ (A\_27d^{A\_27c}))\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E \\ & A\_27a\ A\_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E\ A\_27c \\ & A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E \\ & A\_27a\ A\_27d\ A\_27c\ A\_27b)\ V1abs1)\ V5rep2)))))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x \in A\_27b.(\forall V4y \in \\
& A\_27b.((V3x = V4y) \Leftrightarrow (p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3x))\ (ap\ V2rep\ V4y)))))))))) \\
& \hspace{15em} (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x1 \in A\_27a.(\forall V4x2 \in \\
& A\_27a.(\forall V5y1 \in A\_27a.(\forall V6y2 \in A\_27a.(((p\ (ap\ (ap\ V0R \\
& V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2)))))))))) \\
& \hspace{15em} (32)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\
& \quad (2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\
& (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in \\
& (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (\forall V6f \in (A\_27d^{A\_27c}). \\
& ((\lambda V7x \in A\_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E \\
& \quad A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A\_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x)))))))))) \\
& \hspace{15em} (33)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0REL \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0REL)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x1 \in A\_27a.(\forall V4x2 \in \\
& A\_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2)))))))))) \\
& \hspace{15em} (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27b}).((p\ ( \\
& ap\ (c\_2Ebool\_2E\_21\ A\_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL \\
& \quad A\_27a)\ (ap\ (c\_2Equotient\_2Erespects\ A\_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c\_2Equotient\_2E\_2D\_2D\_3E\ A\_27a\ 2\ A\_27b\ 2)\ V1abs)\ (c\_2Ecombin\_2EI \\
& \quad 2))\ V3f))))))))) \\
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27a}).(\forall V4g \in \\
& \quad (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E\ A\_27a \\
& \quad 2)\ V0R)\ (c\_2Emin\_2E\_3D\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL \\
& \quad A\_27a)\ (ap\ (c\_2Equotient\_2Erespects\ A\_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ ( \\
& ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL\ A\_27a)\ (ap\ (c\_2Equotient\_2Erespects \\
& \quad A\_27a\ 2)\ V0R))\ V4g))))))))) \\
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\
& \quad (2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\
& \quad (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in \\
& \quad (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27b^{A\_27a}). \\
& \quad (\forall V7g \in (A\_27b^{A\_27a}).(\forall V8x \in A\_27a.(\forall V9y \in \\
& \quad A\_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E\ A\_27a \\
& \quad A\_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ ( \\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))) \\
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0E \in ((2^{A\_27a})^{A\_27a}). \\
& \quad (\forall V1P \in (2^{A\_27a}).((p\ (ap\ (c\_2Equotient\_2EEQUIV\ A\_27a) \\
& V0E)) \Rightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL\ A\_27a)\ (ap\ (c\_2Equotient\_2Erespects \\
& \quad A\_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c\_2Ebool\_2E\_21\ A\_27a)\ V1P)))))) \\
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)). \\
& (\forall V1q \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)). \\
& ((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_eq\ V0p)\ V1q)) \Leftrightarrow ((ap\ c\_2Erealax\_2Etreal\_eq \\
& \quad V0p) = (ap\ c\_2Erealax\_2Etreal\_eq\ V1q))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)). \\
& (\forall V1y \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)). \\
& (\forall V2z \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)). \\
& (((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_lt\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_lt \\
& \quad V1y)\ V2z))) \Rightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_lt\ V0x)\ V2z))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0h \in ty\_2Ehreal\_2Ehreal. ((ap\ c\_2Erealax\_2Ehreal\_of\_treal \\
& \quad (ap\ c\_2Erealax\_2Etreal\_of\_hreal\ V0h) = V0h)) \wedge (\forall V1r \in \\
& \quad (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal). \\
& \quad (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_lt\ c\_2Erealax\_2Etreal\_0)\ V1r))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_eq\ (ap\ c\_2Erealax\_2Etreal\_of\_hreal \\
& \quad (ap\ c\_2Erealax\_2Ehreal\_of\_treal\ V1r)))\ V1r))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)). \\
& (\forall V1i \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)). \\
& ((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_eq\ V0h)\ V1i)) \Rightarrow ((ap\ c\_2Erealax\_2Ehreal\_of\_treal \\
& \quad V0h) = (ap\ c\_2Erealax\_2Ehreal\_of\_treal\ V1i))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)). \\
& (\forall V1x2 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)). \\
& (\forall V2y1 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)). \\
& (\forall V3y2 \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)). \\
& (((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_eq \\
& \quad V2y1)\ V3y2))) \Rightarrow ((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_lt\ V0x1)\ V2y1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_lt\ V1x2)\ V3y2))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\
& \quad ty\_2Ehreal\_2Ehreal)\ ty\_2Erealax\_2Ereal)\ c\_2Erealax\_2Etreal\_eq) \\
& \quad c\_2Erealax\_2Ereal\_ABS)\ c\_2Erealax\_2Ereal\_REP))
\end{aligned} \tag{44}$$



Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty\_2Ehreal\_2Ehreal. (\forall V1i \in ty\_2Ehreal\_2Ehreal. \\
& ((p (ap (ap c\_2Ehreal\_2Ehreal\_lt V0h) V1i)) \Leftrightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Erealax\_2Ereal\_of\_hreal V0h)) (ap c\_2Erealax\_2Ereal\_of\_hreal \\
& V1i))))))
\end{aligned} \tag{45}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Erealax\_2Ereal}). ((\forall V1x \in ty\_2Erealax\_2Ereal. \\
& ((p (ap V0P V1x)) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt c\_2Erealax\_2Ereal\_0) \\
& V1x)))) \wedge ((\exists V2x \in ty\_2Erealax\_2Ereal. (p (ap V0P V2x))) \wedge \\
& (\exists V3z \in ty\_2Erealax\_2Ereal. (\forall V4x \in ty\_2Erealax\_2Ereal. \\
& ((p (ap V0P V4x)) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt V4x) V3z)))))) \Rightarrow \\
& ((\exists V5X \in ty\_2Ehreal\_2Ehreal. (p (ap (\lambda V6h \in ty\_2Ehreal\_2Ehreal. \\
& (ap V0P (ap c\_2Erealax\_2Ereal\_of\_hreal V6h))) V5X))) \wedge (\exists V7Y \in \\
& ty\_2Ehreal\_2Ehreal. (\forall V8X \in ty\_2Ehreal\_2Ehreal. ((p (ap \\
& (\lambda V9h \in ty\_2Ehreal\_2Ehreal. (ap V0P (ap c\_2Erealax\_2Ereal\_of\_hreal \\
& V9h))) V8X)) \Rightarrow (p (ap (ap c\_2Ehreal\_2Ehreal\_lt V8X) V7Y))))))
\end{aligned}$$