thm_2Erealax_2ETREAL__10 (TMSBuexd7nyxVMjmxjmvxzc5eJctHFiUcZy)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in 2$

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E$\ V0t)$ c_2Ebool_2EF

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 8 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{4}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{5}$$

Definition 9 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Definition 10 *We define* c_2Ehrat_2Etrat__1 *to be* $(ap\ (ap\ ($c_2Epair_2E_2C $ty\_2Enum\_2Enum\ ty\_2Enum\_$

Let $c\_2Ehrat\_2Etrat\_\_eq : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \tag{6}$$

Let $ty\_2Ehrat\_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehrat\_2Ehrat \tag{7}$$

Let $c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS \in (ty\_2Ehrat\_2Ehrat^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}) \tag{8}$$

Definition 11 *We define* c_2Ehrat_2Ehrat__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 12 *We define* c_2Ehrat_2Ehrat__1 *to be* $(ap$ c_2Ehrat_2Ehrat__ABS c_2Ehrat_2Etrat__1$)$.

Let $c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehra} \tag{9}$$

Definition 13 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 14 *We define* c_2Ehrat_2Ehrat__REP *to be* $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap\ ($c_2Emin_2E_40 $(ty\_2$

Let $c\_2Ehrat\_2Etrat\_\_add : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty} \tag{10}$$

Definition 15 *We define* c_2Ehrat_2Ehrat__add *to be* $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2E$

Definition 16 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 17 *We define* c_2Ehreal_2Ehrat__lt *to be* $\lambda V0x \in ty\_2Ehrat\_2Ehrat.\lambda V1y \in ty\_2Ehrat\_2Ehrat$

Definition 18 *We define* c_2Ehreal_2Ecut__of__hrat *to be* $\lambda V0x \in ty\_2Ehrat\_2Ehrat.(\lambda V1y \in ty\_2Ehrat\_2$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{11}$$

Let $c\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$c\_2Ehreal\_2Ehreal \in (ty\_2Ehreal\_2Ehreal^{(2^{ty\_2Ehrat\_2Ehrat})}) \tag{12}$$

Definition 19 *We define* c_2Ehreal_2Ehreal__1 *to be* $(ap\ c\_2Ehreal\_2Ehreal\ (ap$ c_2Ehreal_2Ecut__of__hrat

Definition 20 *We define* c_2Erealax_2Etreal__0 *to be* $(ap\ (ap$ (c_2Epair_2E_2C $ty\_2Ehreal\_2Ehreal\ ty\_2Eh$

Let $c\_2Ehreal\_2Ecut : \iota$ be given. Assume the following.

$$c\_2Ehreal\_2Ecut \in ((2^{ty\_2Ehrat\_2Ehrat})^{ty\_2Ehreal\_2Ehreal}) \tag{13}$$

Definition 21 *We define* c_2Ehreal_2Ehreal__add *to be* $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2$

Definition 22 *We define* c_2Erealax_2Etreal__1 *to be* $(ap\ (ap$ (c_2Epair_2E_2C $ty\_2Ehreal\_2Ehreal\ ty\_2Eh$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{14}$$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{17}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge((\neg False)\Leftrightarrow True))) \tag{18}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.(\neg((ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V0X)\ V1Y) = V0X)))) \tag{19}$$

Assume the following.

$$(\forall V0x1 \in ty\_2Ehreal\_2Ehreal.(\forall V1y1 \in ty\_2Ehreal\_2Ehreal.(\forall V2x2 \in ty\_2Ehreal\_2Ehreal.(\forall V3y2 \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_\_eq\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)\ V0x1)\ V1y1))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)\ V2x2)\ V3y2)))\Leftrightarrow((ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V0x1)\ V3y2) = (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V2x2)\ V1y1))))))) \tag{20}$$

Theorem 1

$$(\neg(p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_\_eq\ \mathsf{c\_2Erealax\_2Etreal\_\_1})\ \mathsf{c\_2Erealax\_2Etreal\_\_0})))$$