

thm_2Erealax_2ETREAL_ADD_LID
(TMQVCSxaikA42rwqGFXsvfGzwziFTNf9roP)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \tag{2}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \tag{3}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{4}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (6)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (7)$$

Definition 9 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Definition 10 We define $c_2Ehrat_2Etrat_1$ to be $(ap\ (ap\ (c_2Epair_2E2C\ ty_2Enum_2Enum\ ty_2Enum_2Enum_2Enum_2Enum))$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum_2Enum_2Enum)}) \quad (8)$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehrat_2Ehrat \quad (9)$$

Let $c_2Ehrat_2Ehrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_ABS_CLASS \in (ty_2Ehrat_2Ehrat^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})}) \quad (10)$$

Definition 11 We define $c_2Ehrat_2Ehrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum_2Enum_2Enum)$

Definition 12 We define $c_2Ehrat_2Ehrat_1$ to be $(ap\ c_2Ehrat_2Ehrat_ABS\ c_2Ehrat_2Etrat_1)$.

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat_ABS}) \quad (11)$$

Definition 13 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p\ (ap\ P\ x))$ then $(the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 14 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap\ (c_2Emin_2E40\ (ty_2Ehrat_2Ehrat_REP_CLASS\ a)))$

Let $c_2Ehrat_2Etrat_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Epair_2Eprod\ ty_2Enum_2Enum_2Enum_2Enum})^{ty_2Epair_2Eprod\ ty_2Enum_2Enum_2Enum_2Enum}) \quad (12)$$

Definition 15 We define $c_Eh_rat_Eh_rat_add$ to be $\lambda V0T1 \in ty_Eh_rat_Eh_rat.\lambda V1T2 \in ty_Eh_rat_Eh_rat$

Definition 16 We define $c_Ebool_E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_Emin_E_40$

Definition 17 We define $c_Eh_real_Eh_rat_lt$ to be $\lambda V0x \in ty_Eh_rat_Eh_rat.\lambda V1y \in ty_Eh_rat_Eh_rat$

Definition 18 We define $c_Eh_real_Ecut_of_hrat$ to be $\lambda V0x \in ty_Eh_rat_Eh_rat.(\lambda V1y \in ty_Eh_rat_Eh_rat$

Let $ty_Eh_real_Eh_real : \iota$ be given. Assume the following.

$$nonempty\ ty_Eh_real_Eh_real \quad (13)$$

Let $c_Eh_real_Eh_real : \iota$ be given. Assume the following.

$$c_Eh_real_Eh_real \in (ty_Eh_real_Eh_real^{(2^{ty_Eh_rat_Eh_rat})}) \quad (14)$$

Definition 19 We define $c_Eh_real_Eh_real_1$ to be $(ap\ c_Eh_real_Eh_real\ (ap\ c_Eh_real_Ecut_of_hrat$

Definition 20 We define $c_Erealax_Etreax_0$ to be $(ap\ (ap\ (c_Epair_E_2C\ ty_Eh_real_Eh_real\ ty_Eh_real$

Let $c_Erealax_Etreax_add : \iota$ be given. Assume the following.

$$c_Erealax_Etreax_add \in (((ty_Epair_Eprod\ ty_Eh_real_Eh_real\ ty_Eh_real_Eh_real)(ty_Epair_Eprod\ ty_Eh_real_Eh_real\ ty_Eh_real_Eh_real)(ty_Epair_Eprod\ ty_Eh_real_Eh_real)) \quad (15)$$

Let $c_Eh_real_Ecut : \iota$ be given. Assume the following.

$$c_Eh_real_Ecut \in ((2^{ty_Eh_rat_Eh_rat})^{ty_Eh_real_Eh_real}) \quad (16)$$

Definition 21 We define $c_Eh_real_Eh_real_add$ to be $\lambda V0X \in ty_Eh_real_Eh_real.\lambda V1Y \in ty_Eh_real_Eh_real$

Let $c_Erealax_Etreax_eq : \iota$ be given. Assume the following.

$$c_Erealax_Etreax_eq \in ((2^{(ty_Epair_Eprod\ ty_Eh_real_Eh_real\ ty_Eh_real_Eh_real)})^{(ty_Epair_Eprod\ ty_Eh_real_Eh_real)}) \quad (17)$$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \quad (20) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0X \in ty_2Ehreal_2Ehreal. (\forall V1Y \in ty_2Ehreal_2Ehreal. \\
& ((ap (ap c_2Ehreal_2Ehreal_add V0X) V1Y) = (ap (ap c_2Ehreal_2Ehreal_add \\
& V1Y) V0X))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0X \in ty_2Ehreal_2Ehreal. (\forall V1Y \in ty_2Ehreal_2Ehreal. \\
& (\forall V2Z \in ty_2Ehreal_2Ehreal. ((ap (ap c_2Ehreal_2Ehreal_add \\
& V0X) (ap (ap c_2Ehreal_2Ehreal_add V1Y) V2Z)) = (ap (ap c_2Ehreal_2Ehreal_add \\
& (ap (ap c_2Ehreal_2Ehreal_add V0X) V1Y)) V2Z))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0x \in (ty_2Epair_2Eprod A_27a A_27b). ((ap (ap (c_2Epair_2E_2C \\
& A_27a A_27b) (ap (c_2Epair_2EFST A_27a A_27b) V0x)) (ap (c_2Epair_2ESND \\
& A_27a A_27b) V0x)) = V0x))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Ehreal_2Ehreal. (\forall V1y \in ty_2Ehreal_2Ehreal. \\
& (\forall V2z \in ty_2Ehreal_2Ehreal. (((ap (ap c_2Ehreal_2Ehreal_add \\
& V0x) V1y) = (ap (ap c_2Ehreal_2Ehreal_add V0x) V2z)) \Leftrightarrow (V1y = V2z))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in ty_2Ehreal_2Ehreal. (\forall V1y1 \in ty_2Ehreal_2Ehreal. \\
& (\forall V2x2 \in ty_2Ehreal_2Ehreal. (\forall V3y2 \in ty_2Ehreal_2Ehreal. \\
& ((ap (ap c_2Erealax_2Etreal_add (ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal \\
& ty_2Ehreal_2Ehreal) V0x1) V1y1)) (ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal \\
& ty_2Ehreal_2Ehreal) V2x2) V3y2)) = (ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal \\
& ty_2Ehreal_2Ehreal) (ap (ap c_2Ehreal_2Ehreal_add V0x1) V2x2)) \\
& (ap (ap c_2Ehreal_2Ehreal_add V1y1) V3y2))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in ty_2Ehreal_2Ehreal. (\forall V1y1 \in ty_2Ehreal_2Ehreal. \\
& (\forall V2x2 \in ty_2Ehreal_2Ehreal. (\forall V3y2 \in ty_2Ehreal_2Ehreal. \\
& ((p (ap (ap c_2Erealax_2Etreal_eq (ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal \\
& ty_2Ehreal_2Ehreal) V0x1) V1y1)) (ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal \\
& ty_2Ehreal_2Ehreal) V2x2) V3y2))) \Leftrightarrow ((ap (ap c_2Ehreal_2Ehreal_add \\
& V0x1) V3y2) = (ap (ap c_2Ehreal_2Ehreal_add V2x2) V1y1))))))
\end{aligned} \tag{26}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal). \\
& (p (ap (ap c_2Erealax_2Etreal_eq (ap (ap c_2Erealax_2Etreal_add \\
& c_2Erealax_2Etreal_0) V0x)) V0x)))
\end{aligned}$$