

thm_2Erealax_2ETREAL__EQ__SYM (TMSNF- bLJE1g9jWKEa2tE3f6tH3vMhTYNUdM)

October 26, 2020

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (1)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2ESND\ A.27a\ A.27b \in (A.27b^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}) \quad (2)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2EFST\ A.27a\ A.27b \in (A.27a^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}) \quad (3)$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehrat_2Ehrat \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Ehreal_2Ecut : \iota$ be given. Assume the following.

$$c_2Ehreal_2Ecut \in ((2^{ty_2Ehrat_2Ehrat})^{ty_2Ehreal_2Ehreal}) \quad (6)$$

Definition 1 We define $c_2Emin_2E_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b). ((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V0x))\ (ap\ (c_2Epair_2ESND\ A_27a\ A_27b)\ V0x)) = V0x))) \quad (16)$$

Assume the following.

$$(\forall V0x1 \in ty_2Ehreal_2Ehreal. (\forall V1y1 \in ty_2Ehreal_2Ehreal. (\forall V2x2 \in ty_2Ehreal_2Ehreal. (\forall V3y2 \in ty_2Ehreal_2Ehreal. ((p\ (ap\ (ap\ c_2Erealax_2Etreal_eq\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ V0x1)\ V1y1))\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ V2x2)\ V3y2))) \Leftrightarrow ((ap\ (ap\ c_2Ehreal_2Ehreal_add\ V0x1)\ V3y2) = (ap\ (ap\ c_2Ehreal_2Ehreal_add\ V2x2)\ V1y1))))))) \quad (17)$$

Theorem 1

$$(\forall V0x \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal). (\forall V1y \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal). ((p\ (ap\ (ap\ c_2Erealax_2Etreal_eq\ V0x)\ V1y)) \Leftrightarrow (p\ (ap\ (ap\ c_2Erealax_2Etreal_eq\ V1y)\ V0x))))))$$