thm_2Erealax_2ETREAL__ISO (TMRhsdQQjguH9AvAVTmrsqF4jjzFDZHpwXi)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Ehrat\_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehrat\_2Ehrat \tag{1}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{2}$$

Let $c\_2Ehreal\_2Ecut : \iota$ be given. Assume the following.

$$c\_2Ehreal\_2Ecut \in ((2^{ty\_2Ehrat\_2Ehrat})^{ty\_2Ehreal\_2Ehreal}) \tag{3}$$

Definition 8 *We define* c_2Ehreal_2Ehreal__lt *to be* $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2Eh$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{4}$$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{5}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{6}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{7}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{8}$$

Definition 9 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Epair\_2EABS\_\_prod : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{9}$$

Definition 10 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 11 *We define* c_2Ehrat_2Etrat__1 *to be* $(ap\ (ap\ ($c_2Epair_2E_2C $ty\_2Enum\_2Enum\ ty\_2Enum\_$

Let $c\_2Ehrat\_2Etrat\_\_eq : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \tag{10}$$

Let $c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS \in (ty\_2Ehrat\_2Ehrat^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}) \tag{11}$$

Definition 12 *We define* c_2Ehrat_2Ehrat__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 13 *We define* c_2Ehrat_2Ehrat__1 *to be* $(ap$ c_2Ehrat_2Ehrat__ABS c_2Ehrat_2Etrat__1$)$.

Let $c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehra} \tag{12}$$

Definition 14 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota\Rightarrow\iota$.

Definition 15 *We define* c_2Ehrat_2Ehrat__REP *to be* $\lambda V0a \in ty\_2Ehrat\_2Ehrat.(ap$ (c_2Emin_2E_40 $(ty\_2$

Let $c\_2Ehrat\_2Etrat\_\_add : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty} \tag{13}$$

Definition 16 *We define* c_2Ehrat_2Ehrat__add *to be* $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2E$

Definition 17 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 18 *We define* c_2Ehreal_2Ehrat__lt *to be* $\lambda V0x \in ty\_2Ehrat\_2Ehrat.\lambda V1y \in ty\_2Ehrat\_2Ehrat$

Definition 19 *We define* c_2Ehreal_2Ecut__of__hrat *to be* $\lambda V0x \in ty\_2Ehrat\_2Ehrat.(\lambda V1y \in ty\_2Ehrat\_2$

Let $c\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$c\_2Ehreal\_2Ehreal \in (ty\_2Ehreal\_2Ehreal^{(2^{ty\_2Ehrat\_2Ehrat})}) \tag{14}$$

Definition 20 *We define* c_2Ehreal_2Ehreal__1 *to be* $(ap\ c\_2Ehreal\_2Ehreal\ (ap\ \mathsf{c\_2Ehreal\_2Ecut\_\_of\_\_hrat}$

Definition 21 *We define* c_2Ehreal_2Ehreal__add *to be* $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2$

Definition 22 *We define* c_2Erealax_2Etreal__of__hreal *to be* $\lambda V0x \in ty\_2Ehreal\_2Ehreal.(ap\ (ap\ (\mathsf{c\_2Epair}$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{17}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.((ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V0X)\ V1Y) = (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V1Y)\ V0X)))) \tag{18}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.(\forall V2Z \in ty\_2Ehreal\_2Ehreal.((ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V0X)\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V1Y)\ V2Z)) = (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V0X)\ V1Y))\ V2Z))))) \tag{19}$$

Assume the following.

$$(\forall V0x \in ty\_2Ehreal\_2Ehreal.(\forall V1y \in ty\_2Ehreal\_2Ehreal.(\forall V2z \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V0x)\ V1y))\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V0x)\ V2z)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V1y)\ V2z))))))\tag{20}$$

Assume the following.

$$(\forall V0x1 \in ty\_2Ehreal\_2Ehreal.(\forall V1y1 \in ty\_2Ehreal\_2Ehreal.(\forall V2x2 \in ty\_2Ehreal\_2Ehreal.(\forall V3y2 \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_\_lt\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)\ V0x1)\ V1y1))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)\ V2x2)\ V3y2)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V0x1)\ V3y2))\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V2x2)\ V1y1))))))))\tag{21}$$

Theorem 1

$$(\forall V0h \in ty\_2Ehreal\_2Ehreal.(\forall V1i \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V0h)\ V1i))\Rightarrow(p\ (ap\ (ap\ c\_2Erealax\_2Etreal\_\_lt\ (ap\ \mathsf{c\_2Erealax\_2Etreal\_\_of\_\_hreal}\ V0h))\ (ap\ \mathsf{c\_2Erealax\_2Etreal\_\_of\_\_hreal}\ V1i))))))$$