

thm_2Erealax_2ETREAL__MUL__LINV
(TMYj1snzGk5eFJeGFx3pSjSs9s7FWLd3xHB)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 7 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \tag{2}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \tag{3}$$

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{4}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{5}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{6}$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{7}$$

Definition 10 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ x\ y)$

Definition 11 We define $c_2Ehrat_2Etrat_1$ to be $(ap\ (ap\ (c_2Epair_2E2C\ ty_2Enum_2Enum\ ty_2Enum_2Enum)\ ty_2Epair_2Eprod\ ty_2Enum_2Enum))$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \tag{8}$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehrat_2Ehrat \tag{9}$$

Let $c_2Ehrat_2Ehrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_ABS_CLASS \in (ty_2Ehrat_2Ehrat^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})}) \tag{10}$$

Definition 12 We define $c_2Ehrat_2Ehrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 13 We define $c_2Ehrat_2Ehrat_1$ to be $(ap\ c_2Ehrat_2Ehrat_ABS\ c_2Ehrat_2Etrat_1)$.

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat_1}) \tag{11}$$

Definition 14 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A)\ P)$ of type $\iota \Rightarrow \iota$.

Definition 15 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap\ (c_2Emin_2E40\ ty_2Enum_2Enum)\ a)$

Let $c_2Ehrat_2Etrat_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})) \quad (12)$$

Definition 16 We define $c_2Ehrat_2Ehtrat_add$ to be $\lambda V0T1 \in ty_2Ehtrat_2Ehtrat.\lambda V1T2 \in ty_2Ehtrat_2Ehtrat$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 18 We define $c_2Ehreal_2Ehtrat_lt$ to be $\lambda V0x \in ty_2Ehtrat_2Ehtrat.\lambda V1y \in ty_2Ehtrat_2Ehtrat$

Definition 19 We define $c_2Ehreal_2Ecut_of_htrat$ to be $\lambda V0x \in ty_2Ehtrat_2Ehtrat.(\lambda V1y \in ty_2Ehtrat_2Ehtrat$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (13)$$

Let $c_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$c_2Ehreal_2Ehreal \in (ty_2Ehreal_2Ehreal^{(2^{ty_2Ehreal_2Ehreal})}) \quad (14)$$

Definition 20 We define $c_2Ehreal_2Ehreal_1$ to be $(ap\ c_2Ehreal_2Ehreal\ (ap\ c_2Ehreal_2Ecut_of_htrat$

Let $c_2Ehreal_2Ecut : \iota$ be given. Assume the following.

$$c_2Ehreal_2Ecut \in ((2^{ty_2Ehreal_2Ehreal})^{ty_2Ehreal_2Ehreal}) \quad (15)$$

Definition 21 We define $c_2Ehreal_2Ehreal_add$ to be $\lambda V0X \in ty_2Ehreal_2Ehreal.\lambda V1Y \in ty_2Ehreal_2Ehreal$

Definition 22 We define $c_2Erealax_2Etreal_1$ to be $(ap\ (ap\ (c_2Epair_2E_2C\ ty_2Ehreal_2Ehreal\ ty_2Ehreal$

Let $c_2Ehtrat_2Etrat_mul : \iota$ be given. Assume the following.

$$c_2Ehtrat_2Etrat_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})) \quad (16)$$

Definition 23 We define $c_2Ehtrat_2Ehtrat_mul$ to be $\lambda V0T1 \in ty_2Ehtrat_2Ehtrat.\lambda V1T2 \in ty_2Ehtrat_2Ehtrat$

Definition 24 We define $c_2Ehreal_2Ehreal_mul$ to be $\lambda V0X \in ty_2Ehreal_2Ehreal.\lambda V1Y \in ty_2Ehreal_2Ehreal$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})) \quad (17)$$

Definition 25 We define $c_2Ehreal_2Ehreal_sub$ to be $\lambda V0Y \in ty_2Ehreal_2Ehreal.\lambda V1X \in ty_2Ehreal_2Ehreal$

Definition 26 We define $c_2Ehreal_2Ehreal_inv$ to be $\lambda V0X \in ty_2Ehreal_2Ehreal.(ap\ c_2Ehreal_2Ehreal$

Definition 27 We define $c_2Ehreal_2Ehreal_lt$ to be $\lambda V0X \in ty_2Ehreal_2Ehreal.\lambda V1Y \in ty_2Ehreal_2Ehreal$

Definition 28 We define $c_2Erealax_2Etreal_0$ to be $(ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)))$

Definition 29 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Let $c_2Erealax_2Etreal_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_inv \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (18)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)) \quad (19)$$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t)))))) \quad (23)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg (p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg (p V0t)))))) \quad (25)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))))) \quad (26)$$

Assume the following.

$$(\forall V0X \in ty_2Ehreal_2Ehreal.(\forall V1Y \in ty_2Ehreal_2Ehreal. ((ap (ap c_2Ehreal_2Ehreal_add V0X) V1Y) = (ap (ap c_2Ehreal_2Ehreal_add V1Y) V0X)))) \quad (27)$$

Assume the following.

$$(\forall V0X \in ty_2Ehreal_2Ehreal.(\forall V1Y \in ty_2Ehreal_2Ehreal. (\forall V2Z \in ty_2Ehreal_2Ehreal.((ap (ap c_2Ehreal_2Ehreal_add V0X) (ap (ap c_2Ehreal_2Ehreal_add V1Y) V2Z)) = (ap (ap c_2Ehreal_2Ehreal_add (ap (ap c_2Ehreal_2Ehreal_add V0X) V1Y)) V2Z)))))) \quad (28)$$

Assume the following.

$$(\forall V0X \in ty_2Ehreal_2Ehreal.(\forall V1Y \in ty_2Ehreal_2Ehreal. (\forall V2Z \in ty_2Ehreal_2Ehreal.((ap (ap c_2Ehreal_2Ehreal_mul V0X) (ap (ap c_2Ehreal_2Ehreal_add V1Y) V2Z)) = (ap (ap c_2Ehreal_2Ehreal_add (ap (ap c_2Ehreal_2Ehreal_mul V0X) V1Y)) (ap (ap c_2Ehreal_2Ehreal_mul V0X) V2Z)))))) \quad (29)$$

Assume the following.

$$(\forall V0X \in ty_2Ehreal_2Ehreal.((ap (ap c_2Ehreal_2Ehreal_mul c_2Ehreal_2Ehreal_1) V0X) = V0X)) \quad (30)$$

Assume the following.

$$(\forall V0X \in ty_2Ehreal_2Ehreal.((ap (ap c_2Ehreal_2Ehreal_mul (ap c_2Ehreal_2Ehreal_inv V0X)) V0X) = c_2Ehreal_2Ehreal_1)) \quad (31)$$

Assume the following.

$$(\forall V0X \in ty_2Ehreal_2Ehreal.(\forall V1Y \in ty_2Ehreal_2Ehreal. ((p (ap (ap c_2Ehreal_2Ehreal_lt V0X) V1Y)) \Rightarrow ((ap (ap c_2Ehreal_2Ehreal_add (ap (ap c_2Ehreal_2Ehreal_sub V1Y) V0X)) V0X) = V1Y)))) \quad (32)$$

Assume the following.

$$(\forall V0X \in ty_2Ehreal_2Ehreal.(\forall V1Y \in ty_2Ehreal_2Ehreal. ((V0X = V1Y) \vee ((p (ap (ap c_2Ehreal_2Ehreal_lt V0X) V1Y)) \vee (p (ap (ap c_2Ehreal_2Ehreal_lt V1Y) V0X)))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0x \in (ty_2Epair_2Eprod A_27a A_27b).((ap (ap (c_2Epair_2E_2C A_27a A_27b) (ap (c_2Epair_2EFST A_27a A_27b) V0x)) (ap (c_2Epair_2ESND A_27a A_27b) V0x)) = V0x)) \quad (34)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Ehreal_2Ehreal. (\forall V1y \in ty_2Ehreal_2Ehreal. \\
& (\forall V2z \in ty_2Ehreal_2Ehreal. ((ap (ap c_2Ehreal_2Ehreal_mul \\
& (ap (ap c_2Ehreal_2Ehreal_add V0x) V1y)) V2z) = (ap (ap c_2Ehreal_2Ehreal_add \\
& (ap (ap c_2Ehreal_2Ehreal_mul V0x) V2z)) (ap (ap c_2Ehreal_2Ehreal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Ehreal_2Ehreal. (\forall V1y \in ty_2Ehreal_2Ehreal. \\
& (\forall V2z \in ty_2Ehreal_2Ehreal. (((ap (ap c_2Ehreal_2Ehreal_add \\
& V0x) V1y) = (ap (ap c_2Ehreal_2Ehreal_add V0x) V2z)) \Leftrightarrow (V1y = V2z))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Ehreal_2Ehreal. (\forall V1y \in ty_2Ehreal_2Ehreal. \\
& ((p (ap (ap c_2Ehreal_2Ehreal_lt V0x) V1y)) \Rightarrow (\neg (p (ap (ap c_2Ehreal_2Ehreal_lt \\
& V1y) V0x)))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in ty_2Ehreal_2Ehreal. (\forall V1y1 \in ty_2Ehreal_2Ehreal. \\
& (\forall V2x2 \in ty_2Ehreal_2Ehreal. (\forall V3y2 \in ty_2Ehreal_2Ehreal. \\
& ((ap (ap c_2Erealax_2Etrealmul (ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal \\
& ty_2Ehreal_2Ehreal) V0x1) V1y1)) (ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal \\
& ty_2Ehreal_2Ehreal) V2x2) V3y2)) = (ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal \\
& ty_2Ehreal_2Ehreal) (ap (ap c_2Ehreal_2Ehreal_add (ap (ap c_2Ehreal_2Ehreal_mul \\
& V0x1) V2x2)) (ap (ap c_2Ehreal_2Ehreal_mul V1y1) V3y2))) (ap (\\
& ap c_2Ehreal_2Ehreal_add (ap (ap c_2Ehreal_2Ehreal_mul V0x1) \\
& V3y2)) (ap (ap c_2Ehreal_2Ehreal_mul V1y1) V2x2))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Ehreal_2Ehreal. (\forall V1y \in ty_2Ehreal_2Ehreal. \\
& ((ap c_2Erealax_2Etrealmul (ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal \\
& ty_2Ehreal_2Ehreal) V0x) V1y)) = (ap (ap (ap (c_2Ebool_2ECOND (\\
& ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (\\
& ap (ap (c_2Emin_2E_3D ty_2Ehreal_2Ehreal) V0x) V1y)) c_2Erealax_2Etrealmul) \\
& (ap (ap (ap (c_2Ebool_2ECOND (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\
& ty_2Ehreal_2Ehreal)) (ap (ap c_2Ehreal_2Ehreal_lt V1y) V0x)) \\
& (ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) \\
& (ap (ap c_2Ehreal_2Ehreal_add (ap c_2Ehreal_2Ehreal_inv (ap \\
& (ap c_2Ehreal_2Ehreal_sub V0x) V1y))) c_2Ehreal_2Ehreal_1)) \\
& c_2Ehreal_2Ehreal_1)) (ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal \\
& ty_2Ehreal_2Ehreal) c_2Ehreal_2Ehreal_1) (ap (ap c_2Ehreal_2Ehreal_add \\
& (ap c_2Ehreal_2Ehreal_inv (ap (ap c_2Ehreal_2Ehreal_sub V1y) \\
& V0x))) c_2Ehreal_2Ehreal_1))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in ty_2Ehreal_2Ehreal. (\forall V1y1 \in ty_2Ehreal_2Ehreal. \\
& (\forall V2x2 \in ty_2Ehreal_2Ehreal. (\forall V3y2 \in ty_2Ehreal_2Ehreal. \\
& ((p (ap (ap c_2Erealax_2Etreal_eq (ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal \\
& ty_2Ehreal_2Ehreal) V0x1) V1y1)) (ap (ap (c_2Epair_2E_2C ty_2Ehreal_2Ehreal \\
& ty_2Ehreal_2Ehreal) V2x2) V3y2))) \Leftrightarrow ((ap (ap c_2Ehreal_2Ehreal_add \\
& V0x1) V3y2) = (ap (ap c_2Ehreal_2Ehreal_add V2x2) V1y1))))))
\end{aligned} \tag{40}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal). \\
& ((\neg(p (ap (ap c_2Erealax_2Etreal_eq V0x) c_2Erealax_2Etreal_0))) \Rightarrow \\
& (p (ap (ap c_2Erealax_2Etreal_eq (ap (ap c_2Erealax_2Etreal_mul \\
& (ap c_2Erealax_2Etreal_inv V0x)) V0x)) c_2Erealax_2Etreal_1))))
\end{aligned}$$