

thm_2Erealax_2ETREAL__NEG__WELLDEF (TMdvtY6wwjafVvP6koej9wrm89dr2pUTgH4)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{1}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{2}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{3}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{4}$$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \tag{5}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{6}$$

Let $c_2Ehreal_2Ecut : \iota$ be given. Assume the following.

$$c_2Ehreal_2Ecut \in ((2^{ty_2Ehreal_2Ehreal})^{ty_2Ehreal_2Ehreal}) \tag{7}$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0X \in ty_2Ehreal_2Ehreal. (\forall V1Y \in ty_2Ehreal_2Ehreal. \\ & ((ap (ap (c_2Ehreal_2Ehreal_add\ V0X)\ V1Y) = (ap (ap (c_2Ehreal_2Ehreal_add\ V1Y)\ V0X)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod\ A.27a\ A.27b). ((ap (ap (c_2Epair_2E_2C\ A.27a\ A.27b) (ap (c_2Epair_2EFST\ A.27a\ A.27b)\ V0x)) (ap (c_2Epair_2ESND\ A.27a\ A.27b)\ V0x)) = V0x)) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Ehreal_2Ehreal. (\forall V1y \in ty_2Ehreal_2Ehreal. \\ & ((ap (c_2Erealax_2Etreal_neg (ap (ap (c_2Epair_2E_2C\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ V0x)\ V1y)) = (ap (ap (c_2Epair_2E_2C\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ V1y)\ V0x)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0x1 \in ty_2Ehreal_2Ehreal. (\forall V1y1 \in ty_2Ehreal_2Ehreal. \\ & (\forall V2x2 \in ty_2Ehreal_2Ehreal. (\forall V3y2 \in ty_2Ehreal_2Ehreal. \\ & ((p (ap (ap (c_2Erealax_2Etreal_eq (ap (ap (c_2Epair_2E_2C\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ V0x1)\ V1y1)) (ap (ap (c_2Epair_2E_2C\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ V2x2)\ V3y2))) \Leftrightarrow ((ap (ap (c_2Ehreal_2Ehreal_add\ V0x1)\ V3y2) = (ap (ap (c_2Ehreal_2Ehreal_add\ V2x2)\ V1y1))))))) \end{aligned} \quad (19)$$

Theorem 1

$$\begin{aligned} & (\forall V0x1 \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal). \\ & (\forall V1x2 \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal). \\ & ((p (ap (ap (c_2Erealax_2Etreal_eq\ V0x1)\ V1x2)) \Rightarrow (p (ap (ap (c_2Erealax_2Etreal_eq \\ & (ap (c_2Erealax_2Etreal_neg\ V0x1)) (ap (c_2Erealax_2Etreal_neg\ V1x2))))))) \end{aligned}$$