

# thm\_2Erealax\_2Ereal\_\_QUOTIENT (TMKpQJ6kZMswQZtuoHdtc5tYsUbCaTgCsjb)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2T` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A-27a}))))$

**Definition 4** We define `c_2Ebool_2E_2F` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V0t \in 2. V0t))$ .

**Definition 5** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_27E` to be  $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D\_3D\_3E } V0t) (\text{c\_2Ebool\_2E\_2F}))))$

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V2t \in 2. V2t))))$

**Definition 8** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p (\text{ap } P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P (\text{ap } (\text{c\_2Emin\_2E\_40 } A))))$

**Definition 10** We define `c_2Equotient_2EPARTIAL__EQUIV` to be  $\lambda A. 27a : \iota. \lambda V0R \in ((2^{A-27a})^{A-27a}). (\text{ap } (\text{c\_2Ebool\_2E\_3F } A))$

**Definition 11** We define `c_2Equotient_2EEQUIV` to be  $\lambda A. 27a : \iota. \lambda V0E \in ((2^{A-27a})^{A-27a}). (\text{ap } (\text{c\_2Ebool\_2E\_3F } A))$

**Definition 12** We define `c_2Equotient_2EQUOTIENT` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0R \in ((2^{A-27a})^{A-27a}). (\text{ap } (\text{c\_2Equotient\_2EPARTIAL__EQUIV } A))$

Let `ty_2Ehreal_2Ehreal` :  $\iota$  be given. Assume the following.

$$\text{nonempty } \text{ty\_2Ehreal\_2Ehreal} \tag{1}$$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty\_2Epair\_2Eprod } A0 A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (3)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (4)$$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ t))$

Let  $c\_2Erealax\_2Ereal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (5)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (6)$$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& p V0t))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0R \in ((2^{A.27a})^{A.27a}). \\
& ((p (ap (c.2Equotient.2EEQUIV A.27a) V0R)) \Rightarrow (p (ap (c.2Equotient.2EPARTIAL\_EQUIV \\
& A.27a) V0R))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty.2Epair.2Eprod ty.2Ehreal.2Ehreal ty.2Ehreal.2Ehreal). \\
& (\forall V1q \in (ty.2Epair.2Eprod ty.2Ehreal.2Ehreal ty.2Ehreal.2Ehreal). \\
& ((p (ap (ap c.2Erealax.2Etreal\_eq V0p) V1q)) \Leftrightarrow ((ap c.2Erealax.2Etreal\_eq \\
& V0p) = (ap c.2Erealax.2Etreal\_eq V1q))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0a \in ty.2Erealax.2Ereal.((ap c.2Erealax.2Ereal\_ABS\_CLASS \\
& (ap c.2Erealax.2Ereal\_REP\_CLASS V0a) = V0a)) \wedge (\forall V1c \in \\
& (2^{(ty.2Epair.2Eprod ty.2Ehreal.2Ehreal ty.2Ehreal.2Ehreal)}). \\
& ((\exists V2r \in (ty.2Epair.2Eprod ty.2Ehreal.2Ehreal ty.2Ehreal.2Ehreal). \\
& ((p (ap (ap c.2Erealax.2Etreal\_eq V2r) V2r)) \wedge (V1c = (ap c.2Erealax.2Etreal\_eq \\
& V2r)))) \Leftrightarrow ((ap c.2Erealax.2Ereal\_REP\_CLASS (ap c.2Erealax.2Ereal\_ABS\_CLASS \\
& V1c) = V1c))))))
\end{aligned} \tag{17}$$

**Theorem 1**

$$\begin{aligned}
& (p (ap (ap (ap (ap (c.2Equotient.2EQUOTIENT (ty.2Epair.2Eprod ty.2Ehreal.2Ehreal \\
& ty.2Ehreal.2Ehreal) ty.2Erealax.2Ereal) c.2Erealax.2Etreal\_eq) \\
& c.2Erealax.2Ereal\_ABS) c.2Erealax.2Ereal\_REP))
\end{aligned}$$