thm_2Erelation_2EEQC__IDEM (TMRmafLZKiFtJ17h5opxq5gt9R6hNYcWVUJ)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF$

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Definition 8 *We define* c_2Erelation_2ESC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1x \in A\_27a.\lambda V2y \in$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Definition 10 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 11 *We define* c_2Erelation_2ERC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1x \in A\_27a.\lambda V2y$

Definition 12 *We define* c_2Erelation_2EEQC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (c\_2Erelation\_2E$

Definition 13 *We define* c_2Erelation_2Esymmetric *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (c\_2Ebool\_$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{2}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{3}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(p\ (ap\ (\mathsf{c\_2Erelation\_2Esymmetric}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ESC}\ A\_27a)\ V0R)))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Erelation\_2Esymmetric}\ A\_27a)\ V0R)) \Rightarrow ((ap\ (\mathsf{c\_2Erelation\_2ESC}\ A\_27a)\ V0R) = V0R))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).((ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)) = (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(((ap\ (\mathsf{c\_2Erelation\_2ESC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ V0R)) = (ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ESC}\ A\_27a)\ V0R))) \wedge (((ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ V0R)) = (ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ V0R)) \wedge ((ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ V0R)) = (ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Erelation\_2Esymmetric}\ A\_27a)\ V0R)) \Rightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2Esymmetric}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R))))) \tag{9}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).((ap\ (\mathsf{c\_2Erelation\_2EEQC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2EEQC}\ A\_27a)\ V0R)) = (ap\ (\mathsf{c\_2Erelation\_2EEQC}\ A\_27a)\ V0R)))$$