

thm_2Erelation_2EEQC_R (TMNkaP2CuUc995yC2vMG6pDzJXrDGBfJNzA)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1x \in A_27a.V1x)) (\lambda V2y \in A_27a.V2y)))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (P \Rightarrow Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 8 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 9 We define $c_2Erelation_2ESC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).\lambda V1x \in A_27a.\lambda V2y \in A_27a.V2y$

Definition 10 We define $c_2Erelation_2ERC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).\lambda V1x \in A_27a.\lambda V2y \in A_27a.V2y$

Definition 11 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).\lambda V1a \in A_27a.\lambda V2b \in A_27a.V2b$

Definition 12 We define $c_2Erelation_2EEQC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).(ap (c_2Erelation_2ETC$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{2}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (3)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))) \quad (4)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (5)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (6)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1x \in A_27a. (\forall V2y \in A_27a. ((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Rightarrow (p\ (ap\ (ap\ (c_2Erelation_2ETC\ A_27a)\ V0R)\ V1x)\ V2y)))))) \quad (7)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1x \in A_27a. (\forall V2y \in A_27a. ((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Erelation_2EEQC\ A_27a)\ V0R)\ V1x)\ V2y))))))$$