

# thm\_2Erelation\_2EINDUCTION\_\_WF\_\_THM (TMUjsqNfqB2tL2ckpKekEyiMd4YMr6aMNHV)

October 26, 2020

**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2T` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

**Definition 3** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A \cdot 27a}). (\text{ap } V 0P (\text{ap } (\text{c\_2Emin\_2E\_40 } A))))$

**Definition 5** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A \cdot 27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A \cdot 27a}))))$

**Definition 6** We define `c_2Ebool_2E_2F` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V 0t \in 2.V 0t))$ .

**Definition 7** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota$ .

**Definition 8** We define `c_2Ebool_2E_7E` to be  $(\lambda V 0t \in 2. (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D\_3D\_3E } V 0t) (\text{c\_2Ebool\_2E\_2F } 2))))$

**Definition 9** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V 0t1 \in 2. (\lambda V 1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V 2t \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_2F } V 2t) (\text{c\_2Emin\_2E\_3D\_3D\_3E } V 2t))))))$

**Definition 10** We define `c_2Erelation_2EWF` to be  $\lambda A. 27a : \iota. \lambda V 0R \in ((2^{A \cdot 27a})^{A \cdot 27a}). (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V 1t \in 2. (\lambda V 2t \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_2F_5C } V 2t) (\text{c\_2Emin\_2E\_3D\_3D\_3E } V 2t))))))$

**Definition 11** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V 0t1 \in 2. (\lambda V 1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V 2t \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_2F_5C } V 2t) (\text{c\_2Emin\_2E\_3D\_3D\_3E } V 2t))))))$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$(\forall V 0t1 \in 2. (\forall V 1t2 \in 2. (((p \Rightarrow V 0t1) \Rightarrow (p \Rightarrow V 1t2)) \Rightarrow (((p \Rightarrow V 1t2) \Rightarrow (p \Rightarrow V 0t1)) \Rightarrow ((p \Rightarrow V 0t1) \Leftrightarrow (p \Rightarrow V 1t2)))))) \tag{2}$$

Assume the following.

$$((\forall V 0t \in 2. ((\neg(\neg(p \Rightarrow V 0t))) \Leftrightarrow (p \Rightarrow V 0t))) \wedge (((\neg \text{True}) \Leftrightarrow \text{False}) \wedge ((\neg \text{False}) \Leftrightarrow \text{True}))) \tag{3}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (4)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A) \vee \neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A) \wedge \neg(p \ V1B))))))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.(((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \quad (6)$$

Assume the following.

$$(\forall V0A \in 2.(((p \ V0A) \Rightarrow ((\neg(p \ V0A)) \Rightarrow False))) \quad (7)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow (((p \ V0A) \Rightarrow False) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (8)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \ V0A)) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (9)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \ V0A)) \Rightarrow False) \Rightarrow (((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (10)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ((p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg(p \ V2r)) \vee \neg(p \ V1q))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee \neg(p \ V0p))) \wedge ((p \ V2r) \vee ((\neg(p \ V1q)) \vee \neg(p \ V0p)))))))))) \quad (11)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ((p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee \neg(p \ V2r))) \wedge (((p \ V1q) \vee ((\neg(p \ V0p)) \wedge ((p \ V2r) \vee \neg(p \ V0p)))))))))) \quad (12)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ((p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p)))))))) \quad (13)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge ( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee \neg(p \ V0p)))))))))) \quad (14)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q)) \vee \neg(p \ V0p)))))) \quad (15)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& ((\forall V1P \in (2^{A\_27a}). (\forall V2x \in A\_27a. ((\forall V3y \in \\
& A\_27a. ((p \ (ap \ (ap \ V0R \ V3y) \ V2x)) \Rightarrow (p \ (ap \ V1P \ V3y)))) \Rightarrow (p \ (ap \ V1P \ V2x)))) \Rightarrow \\
& (\forall V4x \in A\_27a. (p \ (ap \ V1P \ V4x)))) \Rightarrow (p \ (ap \ (c\_2Erelation\_2EWF \\
& A\_27a) \ V0R))))
\end{aligned}$$