thm_2Erelation_2ERTC__IDEM (TMXbmD-kXRLjouMHNExzPrRL1PkV2pKvMYC2)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Erelation_2ERTC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 7 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b \in$

Definition 8 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Erelation_2ERC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1x \in A\_27a.\lambda V2y \in$

Definition 10 *We define* c_2Erelation_2ESC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1x \in A\_27a.\lambda V2y$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{2}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (((ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)) = (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)) \wedge ((ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ V0R)) = (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). ((ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)) = (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (((ap\ (\mathsf{c\_2Erelation\_2ESC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ V0R)) = (ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ESC}\ A\_27a)\ V0R))) \wedge (((ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ V0R)) = (ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ V0R)) \wedge ((ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ V0R)) = (ap\ (\mathsf{c\_2Erelation\_2ERC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)))))) \tag{7}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). ((ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)) = (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)))$$