

thm_2Erelation_2ERTC__RULES__RIGHT1 (TMT13iM3yd4E48HqURGMBRks9bo13CeQjS6)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 6 We define $c_2Erelation_2ERTC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in A_27a.$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\ & (\forall V1a \in A_27a.(\forall V2b \in A_27a.((p (ap (ap (ap (c_2Erelation_2ERTC \\ & A_27a) V0R) V1a) V2b)) \Leftrightarrow (\forall V3Q \in (2^{A_27a}).(((p (ap V3Q V1a)) \wedge \\ & (\forall V4y \in A_27a.(\forall V5z \in A_27a.(((p (ap V3Q V4y)) \wedge (p (\\ & ap (ap V0R V4y) V5z))) \Rightarrow (p (ap V3Q V5z)))))) \Rightarrow (p (ap V3Q V2b)))))))))) \end{aligned} \quad (1)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\ & ((\forall V1x \in A_27a.(p (ap (ap (ap (c_2Erelation_2ERTC A_27a) \\ & V0R) V1x) V1x))) \wedge (\forall V2x \in A_27a.(\forall V3y \in A_27a.(\forall V4z \in \\ & A_27a.(((p (ap (ap (ap (c_2Erelation_2ERTC A_27a) V0R) V2x) V3y)) \wedge \\ & (p (ap (ap V0R V3y) V4z))) \Rightarrow (p (ap (ap (ap (c_2Erelation_2ERTC A_27a) \\ & V0R) V2x) V4z)))))))))) \end{aligned}$$