thm_2Erelation_2ERTC__lifts__reflexive__transitive__relations (TMJvdU3R3EWWERbGHLYNa9mzmiuQ84sod4F)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Erelation_2Etransitive *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E}$

Definition 7 *We define* c_2Erelation_2Ereflexive *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E}$

Definition 8 *We define* c_2Erelation_2ERTC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 11 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Assume the following.
$$True \tag{1}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{2}$$

1

Assume the following.

$$(\forall V0t \in 2.(((True \land (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \land True) \Leftrightarrow$$
$$(p\ V0t)) \land (((False \land (p\ V0t)) \Leftrightarrow False) \land ((((p\ V0t) \land False) \Leftrightarrow False) \land \quad (3)$$
$$(((p\ V0t) \land (p\ V0t)) \Leftrightarrow (p\ V0t)))))))$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Rightarrow True) \Leftrightarrow$$
$$True) \land (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \land ((\quad (4)$$
$$(p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))))$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \land (((\neg True) \Leftrightarrow False) \land$$
$$((\neg False) \Leftrightarrow True))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow$$
$$(p\ V0t)) \land (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \land (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\quad (6)$$
$$p\ V0t)))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).$$
$$(\forall V1P \in ((2^{A\_27a})^{A\_27a}).(((\forall V2x \in A\_27a.(p\ (ap\ ($$
$$ap\ V1P\ V2x)\ V2x))) \land (\forall V3x \in A\_27a.(\forall V4y \in A\_27a.(\forall V5z \in$$
$$A\_27a.(((p\ (ap\ (ap\ V0R\ V3x)\ V4y)) \land (p\ (ap\ (ap\ V1P\ V4y)\ V5z))) \Rightarrow (p\ (ap$$
$$(ap\ V1P\ V3x)\ V5z)))))))) \Rightarrow (\forall V6x \in A\_27a.(\forall V7y \in A\_27a.$$
$$((p\ (ap\ (ap\ (ap\ (c\_2Erelation\_2ERTC\ A\_27a)\ V0R)\ V6x)\ V7y)) \Rightarrow (p\ (ap$$
$$(ap\ V1P\ V6x)\ V7y)))))))))$$
$$(7)$$

Assume the following.

$$(\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (8)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg (p\ V0A)) \Rightarrow False))) \quad (9)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((p\ V0A) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow$$
$$(((p\ V0A) \Rightarrow False) \Rightarrow ((\neg (p\ V1B)) \Rightarrow False))))) \quad (10)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((\neg (p\ V0A)) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow$$
$$((p\ V0A) \Rightarrow ((\neg (p\ V1B)) \Rightarrow False))))) \quad (11)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \qquad (12)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ($$
$$(p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg($$
$$p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee$$
$$((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))))$$
$$(13)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ($$
$$(p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee$$
$$(\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))))$$
$$(14)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ($$
$$(p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge$$
$$((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))$$
$$(15)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ($$
$$(p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\qquad (16)$$
$$\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee$$
$$(p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))$$
$$(17)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \qquad (18)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \qquad (19)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1Q \in ((2^{A\_27b})^{A\_27b}).$$
$$(\forall V2f \in (A\_27b^{A\_27a}).(((\forall V3x \in A\_27a.(\forall V4y \in$$
$$A\_27a.((p\ (ap\ (ap\ V0R\ V3x)\ V4y)) \Rightarrow (p\ (ap\ (ap\ V1Q\ (ap\ V2f\ V3x))\ (ap\ V2f$$
$$V4y)))))) \wedge ((p\ (ap\ (\mathsf{c\_2Erelation\_2Ereflexive}\ A\_27b)\ V1Q)) \wedge (p$$
$$(ap\ (\mathsf{c\_2Erelation\_2Etransitive}\ A\_27b)\ V1Q)))) \Rightarrow (\forall V5x \in$$
$$A\_27a.(\forall V6y \in A\_27a.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}$$
$$A\_27a)\ V0R)\ V5x)\ V6y)) \Rightarrow (p\ (ap\ (ap\ V1Q\ (ap\ V2f\ V5x))\ (ap\ V2f\ V6y))))))))))))$$