

# thm\_2Erelation\_2ESC\_lifts\_monotonocities (TMZ8jnFosaumcJoertt6igSPivRcxEnb8cw)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2T` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

**Definition 3** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A-27a}))))$

**Definition 5** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V2t \in 2. V2t)))$

**Definition 6** We define `c_2Erelation_2ESC` to be  $\lambda A. 27a : \iota. \lambda V0R \in ((2^{A-27a})^{A-27a}). \lambda V1x \in A. 27a. \lambda V2y \in A. 27a. (\text{c\_2Emin\_2E\_3D_3D_3E } V0R) (\lambda V3z \in A. 27a. V3z)$

**Definition 7** We define `c_2Ebool_2E_2F` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V0t \in 2. V0t)$ .

**Definition 8** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V2t \in 2. V2t)))$

**Definition 9** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D_3D_3E } V0t)) (\text{c\_2Ebool\_2E_2F_5C } V0t)))$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((\text{True} \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow \text{True}) \Leftrightarrow \\ & \text{True}) \wedge (((\text{False} \Rightarrow (p V0t)) \Leftrightarrow \text{True}) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow \text{True}) \wedge (( \\ & (p V0t) \Rightarrow \text{False}) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg (\neg (p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg \text{True}) \Leftrightarrow \text{False}) \wedge \\ & ((\neg \text{False}) \Leftrightarrow \text{True})) \end{aligned} \tag{3}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (4)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (p \ V1B) \wedge (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \wedge ((p \ V0A) \vee (p \ V2C))))))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.(\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \quad (6)$$

Assume the following.

$$(\forall V0A \in 2.((p \ V0A) \Rightarrow (\neg(p \ V0A)) \Rightarrow False)) \quad (7)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow (((p \ V0A) \Rightarrow False) \Rightarrow (\neg(p \ V1B)) \Rightarrow False)))) \quad (8)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p \ V0A)) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow (\neg(p \ V1B)) \Rightarrow False)))) \quad (9)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \ V0A)) \Rightarrow False) \Rightarrow (((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (10)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \Leftrightarrow (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))))))) \quad (11)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \wedge (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p)))))))) \quad (12)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \vee (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \quad (13)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q)) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (14)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q)) \vee \neg(p V0p)))))) \quad (15)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (16)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (17)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\ & (\forall V1f \in (A\_27a^{A\_27a}). ((\forall V2x \in A\_27a. (\forall V3y \in \\ & A\_27a. ((p (ap (ap V0R V2x) V3y)) \Rightarrow (p (ap (ap V0R (ap V1f V2x)) (ap V1f \\ & V3y)))))) \Rightarrow (\forall V4x \in A\_27a. (\forall V5y \in A\_27a. ((p (ap (ap \\ & (ap (c\_2Erelation\_2ESC A\_27a) V0R) V4x) V5y)) \Rightarrow (p (ap (ap (ap (c\_2Erelation\_2ESC \\ & A\_27a) V0R) (ap V1f V4x)) (ap V1f V5y)))))))))) \end{aligned}$$