

thm_2Erelation_2ETC_CASES1
(TMSW5Ywpwbb3TC7cZ7K6Ya7bzCLh4vGLicE)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ecombin_2E_2K` to be $\lambda A. \lambda a : \iota. \lambda A. \lambda b : \iota. (\lambda V0x \in A. \lambda V1y \in A. \lambda V2z \in A. \text{c_2Emin_2E_3D } (V0x = V1y) \wedge V2z = V0x)$

Definition 4 We define `c_2Ecombin_2E_2S` to be $\lambda A. \lambda a : \iota. \lambda A. \lambda b : \iota. \lambda A. \lambda c : \iota. (\lambda V0f \in ((A \rightarrow A)^{A \rightarrow A})) (\lambda V1x \in A. \lambda V2y \in A. \text{c_2Ecombin_2E_2K } (V0f \text{ c_2Emin_2E_3D } (V1x = V2y)))$

Definition 5 We define `c_2Ecombin_2E_2I` to be $\lambda A. \lambda a : \iota. (\text{ap } (\text{ap } (\text{c_2Ecombin_2E_2S } A. \lambda a. \lambda b. \lambda c. \text{c_2Emin_2E_3D } (a = b) \wedge c = a)))$

Definition 6 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P \ x) \text{ then } (\lambda x. x \in A \wedge P \ x) \text{ else } (\lambda x. \text{c_2Emin_2E_3D } (x = x))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define `c_2Ebool_2E_3F` to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A \rightarrow A})) (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } A. \lambda a. \lambda b. \text{c_2Emin_2E_3D } (a = b))))$

Definition 8 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (P \Rightarrow Q)$ of type ι .

Definition 9 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A \rightarrow A})) (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A \rightarrow A}))) (\lambda V1x \in A. \lambda V2y \in A. \text{c_2Emin_2E_3D } (V0P \text{ c_2Emin_2E_3D } (V1x = V2y))))$

Definition 10 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. \text{c_2Emin_2E_3D } (V0t1 = V2t))))$

Definition 11 We define `c_2Erelation_2E_2TC` to be $\lambda A. \lambda a : \iota. \lambda V0R \in ((2^{A \rightarrow A})^{A \rightarrow A}). \lambda V1a \in A. \lambda V2b \in A. \text{c_2Emin_2E_3D } (V0R \text{ c_2Emin_2E_3D } (V1a = V2b))$

Definition 12 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2. V0t)$.

Definition 13 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. \text{c_2Emin_2E_3D } (V0t1 = V2t))))$

Definition 14 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2E_2F})))$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (2)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (3)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (4)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))) \quad (6)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False) \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\exists V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p (ap V0P V2x)))))) \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\ (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ ((\forall V3x \in A_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a.(p\ (\\ ap\ V1Q\ V4x))))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ 2^{A_27a}).((p\ V0P) \vee (\exists V2x \in A_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in \\ A_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ 2^{A_27a}).((\exists V2x \in A_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A_27a.(p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (\\ (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ (p\ V0A)))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ p\ V0A)) \vee (\neg(p\ V1B)))) \wedge (((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0P \in ((2^{A_27b})^{A_27a}).((\forall V1x \in A_27a.(\exists V2y \in \\ A_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A_27b^{A_27a}).(\\ \forall V4x \in A_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((ap\ (c_2Ecombin_2EI \\ A_27a)\ V0x) = V0x)) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\ ((\forall V1x \in A_27a.(\forall V2y \in A_27a.((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Rightarrow \\ (p\ (ap\ (ap\ (ap\ (c_2Erelation_2ETC\ A_27a)\ V0R)\ V1x)\ V2y)))) \wedge (\forall V3x \in \\ A_27a.(\forall V4y \in A_27a.(\forall V5z \in A_27a.(((p\ (ap\ (ap\ (ap \\ (c_2Erelation_2ETC\ A_27a)\ V0R)\ V3x)\ V4y)) \wedge (p\ (ap\ (ap\ (ap\ (c_2Erelation_2ETC \\ A_27a)\ V0R)\ V4y)\ V5z))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Erelation_2ETC\ A_27a) \\ V0R)\ V3x)\ V5z)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0R \in ((2^{A_{.27a}})^{A_{.27a}}). \\ & (\forall V1x \in A_{.27a}. (\forall V2z \in A_{.27a}. ((p (ap (ap (ap (c_2Erelation_2ETC \\ & A_{.27a}) V0R) V1x) V2z)) \Rightarrow ((p (ap (ap V0R V1x) V2z)) \vee (\exists V3y \in A_{.27a}. \\ & ((p (ap (ap V0R V1x) V3y)) \wedge (p (ap (ap (ap (c_2Erelation_2ETC A_{.27a}) \\ & V0R) V3y) V2z)))))))))) \end{aligned} \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ & ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ & ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \end{aligned} \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\ & p V2r) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee \neg(p \ V0p)))))))))) \quad (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q)) \vee \neg(p \ V0p)))))) \quad (31)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& (\forall V1x \in A_27a. (\forall V2z \in A_27a. ((p \ (ap \ (ap \ (ap \ (c_2Erelation_2ETC \\
& A_27a) \ V0R) \ V1x) \ V2z)) \Leftrightarrow ((p \ (ap \ (ap \ V0R \ V1x) \ V2z)) \vee (\exists V3y \in A_27a. \\
& ((p \ (ap \ (ap \ V0R \ V1x) \ V3y)) \wedge (p \ (ap \ (ap \ (ap \ (c_2Erelation_2ETC \ A_27a) \\
& \ V0R) \ V3y) \ V2z))))))))))
\end{aligned}$$