

thm_2Erelation_2EWFREC__COROLLARY (TMTP5FpeMoZGGdcqesfV6f4frxKEJuxb5Hm)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})$

Definition 4 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Let $c_2Ebool_2E_EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2E_EARB A_27a \in A_27a \quad (1)$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then $(\lambda x.x \in A \wedge p)$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_COND$ to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Definition 10 We define $c_2Erelation_2E_ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1M$

Definition 11 We define $c_2Erelation_2E_ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b$

Definition 12 We define $c_2Erelation_2E_Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 13 We define $c_2Erelation_2E_ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 14 We define $c_2Erelation_2E_EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 16 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). (ap\ (c_2Ebool_2E_21$

Assume the following.

$$True \tag{2}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1M \in ((A_27b)^{A_27a})^{(A_27b)^{A_27a}}). \\ & ((p\ (ap\ (c_2Erelation_2EWF\ A_27a)\ V0R)) \Rightarrow (\forall V2x \in A_27a. (\\ & (ap\ (ap\ (ap\ (c_2Erelation_2EWFREC\ A_27a\ A_27b)\ V0R)\ V1M)\ V2x) = (\\ & ap\ (ap\ V1M\ (ap\ (ap\ (ap\ (c_2Erelation_2ERESTRICT\ A_27a\ A_27b)\ (ap \\ & (ap\ (c_2Erelation_2EWFREC\ A_27a\ A_27b)\ V0R)\ V1M))\ V0R)\ V2x)))))) \end{aligned} \tag{4}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0M \in ((A_27b)^{A_27a})^{(A_27b)^{A_27a}}). (\forall V1R \in ((2^{A_27a})^{A_27a}). \\ & (\forall V2f \in (A_27b)^{A_27a}). ((V2f = (ap\ (ap\ (c_2Erelation_2EWFREC \\ & A_27a\ A_27b)\ V1R)\ V0M)) \Rightarrow ((p\ (ap\ (c_2Erelation_2EWF\ A_27a)\ V1R)) \Rightarrow \\ & (\forall V3x \in A_27a. ((ap\ V2f\ V3x) = (ap\ (ap\ V0M\ (ap\ (ap\ (ap\ (c_2Erelation_2ERESTRICT \\ & A_27a\ A_27b)\ V2f)\ V1R)\ V3x))\ V3x)))))) \end{aligned}$$