

thm\_2Erelation\_2EWF\_EQ\_INDUCTION\_THM  
(TMUtJR-  
Wxt35nUbKZAHyo2Qkus3gnHS9PN7F)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})$

**Definition 4** We define  $c\_2Ebool\_2E\_EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_EF$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 10** We define  $c\_2Erelation\_2EWF$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2E\_21$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\ & ((p (ap (c\_2Erelation\_2EWF A\_27a) V0R)) \Rightarrow (\forall V1P \in (2^{A\_27a}). \\ & ((\forall V2x \in A\_27a.((\forall V3y \in A\_27a.((p (ap (ap V0R V3y) V2x)) \Rightarrow \\ & (p (ap V1P V3y)))) \Rightarrow (p (ap V1P V2x)))) \Rightarrow (\forall V4x \in A\_27a.(p (ap \\ & V1P V4x))))))))) \quad (2) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& \quad ((\forall V1P \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((\forall V3y \in \\
& A\_27a. ((p (ap (ap (ap V0R V3y) V2x)) \Rightarrow (p (ap V1P V3y)))) \Rightarrow (p (ap V1P V2x)))) \Rightarrow \\
& \quad (\forall V4x \in A\_27a. (p (ap V1P V4x)))))) \Rightarrow (p (ap (c\_2Erelation\_2EWF \\
& \quad A\_27a) V0R))))))
\end{aligned} \tag{3}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& \quad ((p (ap (c\_2Erelation\_2EWF\ A\_27a) V0R)) \Leftrightarrow (\forall V1P \in (2^{A\_27a}). \\
& \quad ((\forall V2x \in A\_27a. ((\forall V3y \in A\_27a. ((p (ap (ap V0R V3y) V2x)) \Rightarrow \\
& (p (ap V1P V3y)))) \Rightarrow (p (ap V1P V2x)))) \Rightarrow (\forall V4x \in A\_27a. (p (ap \\
& \quad V1P V4x))))))))))
\end{aligned}$$