

# thm\_2Erelation\_2EWF\_\_antisymmetric (TMTxVShUF5Z86M1rfxWYpanfXVzN1kT9Cxx)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2T` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

**Definition 5** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

**Definition 6** We define `c_2Erelation_2Eantisymmetric` to be  $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_21$

**Definition 7** We define `c_2Erelation_2ETC` to be  $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in A_27a$

**Definition 8** We define `c_2Ebool_2E_2F` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 9** We define `c_2Ebool_2E_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

**Definition 10** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 11** We define `c_2Ebool_2E_2F_3F` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

**Definition 12** We define `c_2Erelation_2EWF` to be  $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_21$

**Definition 13** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \tag{2}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (3)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (4)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (6)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A\_27a.(p (ap V1Q V4x))))))) \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p V0P) \wedge (\forall V2x \in A\_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (8)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).((\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p (ap (ap V0R V1x) V2y)) \Rightarrow (p (ap (ap (ap (c.2Erelation\_2ETC A\_27a) V0R) V1x) V2y)))))) \wedge (\forall V3x \in A\_27a.(\forall V4y \in A\_27a.(\forall V5z \in A\_27a.(((p (ap (ap (ap (c.2Erelation\_2ETC A\_27a) V0R) V3x) V4y)) \wedge (p (ap (ap (ap (c.2Erelation\_2ETC A\_27a) V0R) V4y) V5z))) \Rightarrow (p (ap (ap (ap (c.2Erelation\_2ETC A\_27a) V0R) V3x) V5z)))))))))) \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p (ap (c.2Erelation\_2EWF A\_27a) V0R)) \Rightarrow ((p (ap (ap (ap (c.2Erelation\_2ETC A\_27a) V0R) V1x) V2y)) \Rightarrow (\neg(V1x = V2y))))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (11)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (12)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (13)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (14)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (15)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p))))))))))))) \quad (16)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (17)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (18)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (19)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (20)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p)))) \quad (21)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q))))) \quad (22)$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0R \in ((2^{A.27a})^{A.27a}). \\ & ((p \ (ap \ (c.2Erelation.2EWF \ A.27a) \ V0R)) \Rightarrow (p \ (ap \ (c.2Erelation.2Eantisymmetric \\ & \quad A.27a) \ V0R)))) \end{aligned}$$