

thm_2Erelation_2Einv__EQC
(TMZwYYh8aoc3MWvNgoPjiHft5RTrtPmVmUw)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Erelation_2E_2ESC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1x \in A_27a.\lambda V2y \in A_27a$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 9 We define $c_2Erelation_2E_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in A_27a$

Definition 10 We define $c_2Erelation_2E_2ERC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1x \in A_27a.\lambda V2y \in A_27a$

Definition 11 We define $c_2Erelation_2E_2EEQC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Erelation_2E_2ESC$

Definition 12 We define $c_2Erelation_2E_2Einv$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27b})^{A_27a}).\lambda V1x \in A_27a$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{2}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (3)
\end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (4)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& ((ap \ (c_2Erelation_2Einv \ A_27a \ A_27a) \ (ap \ (c_2Erelation_2ERC \\
& A_27a) \ V0R)) = (ap \ (c_2Erelation_2ERC \ A_27a) \ (ap \ (c_2Erelation_2Einv \\
& A_27a \ A_27a) \ V0R)))) \quad (5)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& (((ap \ (c_2Erelation_2Einv \ A_27a \ A_27a) \ (ap \ (c_2Erelation_2ESC \\
& A_27a) \ V0R)) = (ap \ (c_2Erelation_2ESC \ A_27a) \ V0R)) \wedge ((ap \ (c_2Erelation_2ESC \\
& A_27a) \ (ap \ (c_2Erelation_2Einv \ A_27a \ A_27a) \ V0R)) = (ap \ (c_2Erelation_2ESC \\
& A_27a) \ V0R)))) \quad (6)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& ((ap \ (c_2Erelation_2Einv \ A_27a \ A_27a) \ (ap \ (c_2Erelation_2ETC \\
& A_27a) \ V0R)) = (ap \ (c_2Erelation_2ETC \ A_27a) \ (ap \ (c_2Erelation_2Einv \\
& A_27a \ A_27a) \ V0R)))) \quad (7)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& (((ap \ (c_2Erelation_2Einv \ A_27a \ A_27a) \ (ap \ (c_2Erelation_2EEQC \\
& A_27a) \ V0R)) = (ap \ (c_2Erelation_2EEQC \ A_27a) \ V0R)) \wedge ((ap \ (c_2Erelation_2EEQC \\
& A_27a) \ (ap \ (c_2Erelation_2Einv \ A_27a \ A_27a) \ V0R)) = (ap \ (c_2Erelation_2EEQC \\
& A_27a) \ V0R))))
\end{aligned}$$