

# thm\_2Erelation\_2Esymmetric\_TC (TMNTbe- CukDAyyYB8DZy6Bt7NWEgajUejgAa)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2T` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 3** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. if (\exists x \in A. p (ap P x)) then (the (\lambda x. x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define `c_2Ebool_2E_3F` to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40 A_27a P))))$

**Definition 5** We define `c_2Ecombin_2E_K` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

**Definition 6** We define `c_2Ecombin_2E_S` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

**Definition 7** We define `c_2Ecombin_2E_I` to be  $\lambda A_27a : \iota. (ap (ap (c_2Ecombin_2E_S A_27a (A_27a^{A_27a})) A_27a))$

**Definition 8** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a}) P))))$

**Definition 9** We define `c_2Erelation_2E_symmetric` to be  $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). (ap (c_2Ebool_2E_21 A_27a R))$

**Definition 10** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. V2t))))$

**Definition 12** We define `c_2Erelation_2E_ETC` to be  $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1a \in A_27a. \lambda V2b \in A_27b. V0R a b$

**Definition 13** We define `c_2Ebool_2E_EF` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2. V0t))$ .

**Definition 14** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. V2t))))$

**Definition 15** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$

Assume the following.

$$True \quad (1)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (2)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (3)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (4)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))) \quad (6)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (8)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False) \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))) \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in \\ (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ ((\forall V3x \in A\_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A\_27a. (p\ ( \\ ap\ V1Q\ V4x))))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in \\ (2^{A\_27a}). ((\exists V2x \in A\_27a. ((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ ((\exists V3x \in A\_27a. (p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A\_27a. (p\ ( \\ ap\ V1Q\ V4x))))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in ( \\ 2^{A\_27a}). ((p\ V0P) \vee (\exists V2x \in A\_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in \\ A\_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee ( \\ (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ (p\ V0A)))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg( \\ p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow ((p \\ V0t1) \Rightarrow (p\ V1t2)) \wedge ((p\ V1t2) \Rightarrow (p\ V0t1)))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap\ (c.2Ecombin\_2EI \\ A\_27a)\ V0x) = V0x)) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\ ((\forall V1x \in A\_27a. (\forall V2y \in A\_27a. ((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Rightarrow \\ (p\ (ap\ (ap\ (ap\ (c.2Erelation\_2ETC\ A\_27a)\ V0R)\ V1x)\ V2y)))) \wedge (\forall V3x \in \\ A\_27a. (\forall V4y \in A\_27a. (\forall V5z \in A\_27a. (((p\ (ap\ (ap\ (ap \\ (c.2Erelation\_2ETC\ A\_27a)\ V0R)\ V3x)\ V4y)) \wedge (p\ (ap\ (ap\ (ap\ (c.2Erelation\_2ETC \\ A\_27a)\ V0R)\ V4y)\ V5z))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c.2Erelation\_2ETC\ A\_27a) \\ V0R)\ V3x)\ V5z)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& (\forall V1P \in ((2^{A\_27a})^{A\_27a}). (((\forall V2x \in A\_27a. (\forall V3y \in \\
& A\_27a. ((p (ap (ap (ap V0R V2x) V3y)) \Rightarrow (p (ap (ap V1P V2x) V3y)))))) \wedge (\forall V4x \in \\
& A\_27a. (\forall V5y \in A\_27a. (\forall V6z \in A\_27a. (((p (ap (ap V1P \\
& V4x) V5y)) \wedge (p (ap (ap V1P V5y) V6z))) \Rightarrow (p (ap (ap V1P V4x) V6z)))))) \Rightarrow \\
& (\forall V7u \in A\_27a. (\forall V8v \in A\_27a. ((p (ap (ap (ap (c\_2Erelation\_2ETC \\
& A\_27a) V0R) V7u) V8v)) \Rightarrow (p (ap (ap V1P V7u) V8v)))))))))
\end{aligned} \tag{21}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{22}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{25}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\
& (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\
& (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\
& (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r)))) \wedge \\
& ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge ( \\
& \neg(p V1q)) \vee ((p V2r) \vee \neg(p V0p)))))))))) \quad (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q)) \vee \neg(p V0p)))))) \quad (31)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0R \in ((2^{A.27a})^{A.27a}). \\
& ((p (ap (c.2Erelation_2Esymmetric A.27a) V0R)) \Rightarrow (p (ap (c.2Erelation_2Esymmetric \\
& A.27a) (ap (c.2Erelation_2ETC A.27a) V0R))))))
\end{aligned}$$