

thm_2Eres__quan_2EIN__BIGINTER__RES__FORALL (TMGqr6stqAyQ5KWeAnzB28HumTLLmXup8X7)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (3)$$

Definition 8 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set$

Definition 9 We define `c_2Ebool_2ERES_FORALL` to be $\lambda A_{27a} : \iota. (\lambda V0p \in (2^{A-27a}). (\lambda V1m \in (2^{A-27a}). (a$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$\forall A_{27a}. nonempty\ A_{27a} \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_{27a}. (p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{5}$$

Assume the following.

$$\forall A_{27a}. nonempty\ A_{27a} \Rightarrow (\forall V0x \in A_{27a}. ((V0x = V0x) \Leftrightarrow True)) \tag{6}$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty\ A_{27a} \Rightarrow (\forall V0x \in A_{27a}. (\forall V1B \in \\ & (2^{(2^{A-27a})}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{27a})\ V0x)\ (ap\ (c_2Epred_set_2EBIGINTER \\ & A_{27a})\ V1B))) \Leftrightarrow (\forall V2P \in (2^{A-27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & (2^{A-27a})\ V2P)\ V1B)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{27a})\ V0x)\ V2P)))))))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty\ A_{27a} \Rightarrow (\forall V0P \in (2^{A-27a}). (\forall V1f \in \\ & (2^{A-27a}). ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_{27a})\ V0P)\ V1f)) \Leftrightarrow \\ & (\forall V2x \in A_{27a}. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{27a})\ V2x)\ V0P)) \Rightarrow \\ & (p\ (ap\ V1f\ V2x)))))))))) \end{aligned} \tag{8}$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. nonempty\ A_{27a} \Rightarrow (\forall V0x \in A_{27a}. (\forall V1sos \in \\ & (2^{(2^{A-27a})}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{27a})\ V0x)\ (ap\ (c_2Epred_set_2EBIGINTER \\ & A_{27a})\ V1sos))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ (2^{A-27a}) \\ & V1sos)\ (\lambda V2s \in (2^{A-27a}). (ap\ (ap\ (c_2Ebool_2EIN\ A_{27a})\ V0x)\ \\ & V2s)))))))))) \end{aligned}$$