

thm_2Eres__quan_2EIN__BIGUNION__RES__EXISTS
(TMVR-
WoRP885PH13Btgmu5N6cDwh5YC1Jm1t)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 6 We define $c_2Emarker_2EAC$ to be $\lambda V0b1 \in 2.\lambda V1b2 \in 2.(ap (ap c_2Ebool_2E_2F_5C V0b1) V1b2)$

Definition 7 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod A_27a 2)^{A_27b})}) \quad (3)$$

Definition 11 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})})$.

Definition 12 We define $c_2Ebool_2ERES_EXISTS$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (5)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \wedge (p V1t2)) \Leftrightarrow ((p V1t2) \wedge \\ (p V0t1)))))) \quad (6)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge \\ ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\ True)) \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1sos \in \\ (2^{(2^{A_27a})}).((p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (ap (c_2Epred_set_2EBIGUNION \\ A_27a) V1sos))) \Leftrightarrow (\exists V2s \in (2^{A_27a}).((p (ap (ap (c_2Ebool_2EIN \\ A_27a) V0x) V2s)) \wedge (p (ap (ap (c_2Ebool_2EIN (2^{A_27a}) V2s) V1sos))))))) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1f \in \\ (2^{A_27a}).((p (ap (ap (c_2Ebool_2ERES_EXISTS A_27a) V0P) V1f))) \Leftrightarrow \\ (\exists V2x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V0P)) \wedge \\ (p (ap V1f V2x))))))) \quad (10)$$

Theorem 1

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1sos \in \\ (2^{(2^{A_27a})}).((p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (ap (c_2Epred_set_2EBIGUNION \\ A_27a) V1sos))) \Leftrightarrow (p (ap (ap (c_2Ebool_2ERES_EXISTS (2^{A_27a}) \\ V1sos) (\lambda V2s \in (2^{A_27a}).(ap (ap (c_2Ebool_2EIN A_27a) V0x) \\ V2s))))))))))$$