

thm_2Eres__quan_2ERES__EXISTS__ALT
 (TMJpptG-
 BeEotR45iGQW1qBsFH4rHEYZYCE)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
 of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p (ap P x)))$
 of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a) V0P)))$

Definition 5 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$
 of type ι .

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}) V0P) V0P)))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 9 We define $c_2Ebool_2ERES_EXISTS$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).(ap (c_2Emin_2E_40 A_27a) V1m)))$

Definition 10 We define $c_2Ebool_2ERES_SELECT$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).(ap (c_2Emin_2E_40 A_27a) V1m)))$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{2}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V1x) V0P)) \Leftrightarrow (p (ap V0P V1x)))))) \tag{3}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1f \in \\ (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2ERES_EXISTS\ A.27a)\ V0P)\ V1f))) \Leftrightarrow \\ (\exists V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0P)) \wedge \\ (p\ (ap\ V1f\ V2x)))))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1f \in \\ (2^{A.27a}). ((ap\ (ap\ (c.2Ebool.2ERES_SELECT\ A.27a)\ V0P)\ V1f) = \\ (ap\ (c.2Emin.2E.40\ A.27a)\ (\lambda V2x \in A.27a. (ap\ (ap\ c.2Ebool.2E.2F.5C \\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0P))\ (ap\ V1f\ V2x)))))) \end{aligned} \quad (5)$$

Theorem 1

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0p \in (2^{A.27a}). (\forall V1m \in \\ (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2ERES_EXISTS\ A.27a)\ V0p)\ V1m))) \Leftrightarrow \\ ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ (ap\ (ap\ (c.2Ebool.2ERES_SELECT \\ A.27a)\ V0p)\ V1m))\ V0p)) \wedge (p\ (ap\ V1m\ (ap\ (ap\ (c.2Ebool.2ERES_SELECT \\ A.27a)\ V0p)\ V1m)))))) \end{aligned}$$