

thm_2Eres__quan_2ERES__EXISTS__BIGINTER (TMTn8XTrZ2NQtwLrAk68SK2gYE7uEKp8aqm)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_21 2) (c_2Epair_2EABS_prod A_27a A_27b))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (3)$$

Definition 8 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set_2EGSPEC A_27a 2) (c_2Epair_2E_2C A_27a A_27a))$

Definition 9 We define $c_2Ebool_2ERES_FORALL$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).(ap (c_2Ebool_2E_21 2) (c_2Epair_2E_2C A_27a A_27a))))$

Definition 10 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p (ap P x))$ of type $\iota \Rightarrow \iota$).

Definition 11 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 12 We define `c_2Ebool_2ERES_EXISTS` to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A-27a}).(\lambda V1m \in (2^{A-27a}).($

Assume the following.

$$True \tag{4}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{5}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{6}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1B \in (2^{(2^{A-27a})}).((p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (ap (c_2Epred_set_2EBIGINTER A_27a) V1B))) \Leftrightarrow (\forall V2P \in (2^{A-27a}).((p (ap (ap (c_2Ebool_2EIN (2^{A-27a}) V2P) V1B)) \Rightarrow (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) V2P)))))))))) \tag{7}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1f \in (2^{A-27a}).((p (ap (ap (c_2Ebool_2ERES_FORALL A_27a) V0P) V1f)) \Leftrightarrow (\forall V2x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V0P)) \Rightarrow (p (ap V1f V2x)))))))))) \tag{8}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1f \in (2^{A-27a}).((p (ap (ap (c_2Ebool_2ERES_EXISTS A_27a) V0P) V1f)) \Leftrightarrow (\exists V2x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V0P)) \wedge (p (ap V1f V2x)))))))))) \tag{9}$$

Theorem 1

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1sos \in (2^{(2^{A-27a})}).((p (ap (ap (c_2Ebool_2ERES_EXISTS A_27a) (ap (c_2Epred_set_2EBIGINTER A_27a) V1sos)) (\lambda V2x \in A_27a.(ap V0P V2x)))) \Leftrightarrow (\exists V3x \in A_27a.((p (ap (ap (c_2Ebool_2ERES_FORALL (2^{A-27a}) V1sos) (\lambda V4s \in (2^{A-27a}).(ap (ap (c_2Ebool_2EIN A_27a) V3x) V4s)))) \wedge (p (ap V0P V3x))))))))))$$