

thm_2Eres__quan_2ERES__EXISTS__REORDER
(TMT-
TqAawRmhcBwK1kH7AkZMaGsa5tHU7zJK)

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Definition 1 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2EIN` to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 4 We define `c_2Ebool_2EET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a) P)))$

Definition 9 We define `c_2Ebool_2ERES__EXISTS` to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).(ap (c_2Emin_2E_40 A_27a) m)))$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (1)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V1x) V0P)) \Leftrightarrow (p (ap V0P V1x)))))) \quad (2)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1f \in (2^{A_27a}).((p (ap (ap (c_2Ebool_2ERES__EXISTS A_27a) V0P) V1f)) \Leftrightarrow (\exists V2x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V0P)) \wedge (p (ap V1f V2x))))))) \quad (3)$$

Theorem 1

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\\ \forall V0P \in (2^{A_{27a}}).(\forall V1Q \in (2^{A_{27b}}).(\forall V2R \in \\ ((2^{A_{27b}})^{A_{27a}}).((p\ (ap\ (ap\ (c_2Ebool_2ERES_EXISTS\ A_{27a}) \\ V0P)\ (\lambda V3i \in A_{27a}.(ap\ (ap\ (c_2Ebool_2ERES_EXISTS\ A_{27b})\ V1Q) \\ (\lambda V4j \in A_{27b}.(ap\ (ap\ V2R\ V3i)\ V4j)))))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_EXISTS \\ A_{27b})\ V1Q)\ (\lambda V5j \in A_{27b}.(ap\ (ap\ (c_2Ebool_2ERES_EXISTS\ A_{27a}) \\ V0P)\ (\lambda V6i \in A_{27a}.(ap\ (ap\ V2R\ V6i)\ V5j)))))))))) \end{aligned}$$