

thm_2Eres__quan_2ERES__EXISTS__UNIQUE__F (TMVPvqowJC5Jzopck6tZr9EF1aWtfqTXy7u)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_7E` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2E_3F` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 9 We define `c_2Ebool_2E_1N` to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Definition 10 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 11 We define `c_2Ebool_2E_3F_21` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap c_2Ebool_2E_2F_5C$

Definition 12 We define `c_2Ebool_2ERES__FORALL` to be $\lambda A.27a : \iota.(\lambda V0p \in (2^{A-27a}).(\lambda V1m \in (2^{A-27a}).(ap$

Definition 13 We define `c_2Ebool_2ERES__EXISTS` to be $\lambda A.27a : \iota.(\lambda V0p \in (2^{A-27a}).(\lambda V1m \in (2^{A-27a}).(ap$

Definition 14 We define `c_2Ebool_2ERES__EXISTS__UNIQUE` to be $\lambda A.27a : \iota.(\lambda V0p \in (2^{A-27a}).(\lambda V1m \in (2^{A-27a}).(ap$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{2}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (3)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (4)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((p\ (ap \\ & (c_2Ebool_2E_3F_21\ A_27a)\ (\lambda V1x \in A_27a.(ap\ V0P\ V1x)))) \Leftrightarrow ((\\ & \exists V2x \in A_27a.(p\ (ap\ V0P\ V2x))) \wedge (\forall V3x \in A_27a. (\forall V4y \in \\ & A_27a. (((p\ (ap\ V0P\ V3x)) \wedge (p\ (ap\ V0P\ V4y))) \Rightarrow (V3x = V4y)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1s \in \\
& (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2ERES_EXISTS_UNIQUE\ A.27a) \\
& V1s)\ (\lambda V2x \in A.27a.(ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ (c.2Ebool.2E.3F.21 \\
& A.27a)\ (\lambda V3x \in A.27a.(ap\ (ap\ c.2Ebool.2E.2F.5C\ (ap\ (ap\ (c.2Ebool.2EIN \\
& A.27a)\ V3x)\ V1s))\ (ap\ V0P\ V3x)))))))))
\end{aligned} \tag{12}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0P \in A.27a. (\forall V1s \in (2^{A.27b}). (\\
& \forall V2x \in A.27c. (\neg (p\ (ap\ (ap\ (c.2Ebool.2ERES_EXISTS_UNIQUE \\
& A.27b)\ V1s)\ (\lambda V3x \in A.27b.c.2Ebool.2EF)))))))
\end{aligned}$$