

thm\_2Eres\_\_quan\_2ERES\_\_EXISTS\_\_UNIQUE\_\_T  
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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A.27a : \iota. (\lambda V0P \in (2^{A.27a}). (ap (ap (c_2Emin_2E_3D (2^{A.27a}))$

**Definition 4** We define `c_2Ebool_2EF` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define `c_2Ebool_2EIN` to be  $\lambda A.27a : \iota. (\lambda V0x \in A.27a. (\lambda V1f \in (2^{A.27a}). (ap V1f V0x)))$

**Definition 6** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

**Definition 8** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge p (ap P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define `c_2Ebool_2E_3F` to be  $\lambda A.27a : \iota. (\lambda V0P \in (2^{A.27a}). (ap V0P (ap (c_2Emin_2E_40 A.27a) P)))$

**Definition 10** We define `c_2Ebool_2E_3F_21` to be  $\lambda A.27a : \iota. (\lambda V0P \in (2^{A.27a}). (ap (ap c_2Ebool_2E_2F_5C (c_2Emin_2E_40 A.27a) P) P))$

**Definition 11** We define `c_2Ebool_2ERES__FORALL` to be  $\lambda A.27a : \iota. (\lambda V0p \in (2^{A.27a}). (\lambda V1m \in (2^{A.27a}). (ap (c_2Ebool_2E_3F_21 A.27a) (ap V1m V0p))))$

**Definition 12** We define `c_2Ebool_2ERES__EXISTS` to be  $\lambda A.27a : \iota. (\lambda V0p \in (2^{A.27a}). (\lambda V1m \in (2^{A.27a}). (ap (c_2Ebool_2E_3F_21 A.27a) (ap V1m V0p))))$

**Definition 13** We define `c_2Ebool_2ERES__EXISTS__UNIQUE` to be  $\lambda A.27a : \iota. (\lambda V0p \in (2^{A.27a}). (\lambda V1m \in (2^{A.27a}). (ap (c_2Ebool_2E_3F_21 A.27a) (ap V1m V0p))))$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A.27a. nonempty A.27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A.27a. (p V0t)) \Leftrightarrow (p V0t))) \tag{2}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (3)
\end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (4)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1s \in \\
& (2^{A\_27a}).((p \ (ap \ (ap \ (c\_2Ebool\_2ERES\_EXISTS\_UNIQUE \ A\_27a) \\
& V1s) \ (\lambda V2x \in A\_27a.(ap \ V0P \ V2x)))) \Leftrightarrow (p \ (ap \ (c\_2Ebool\_2E\_3F\_21 \\
& A\_27a) \ (\lambda V3x \in A\_27a.(ap \ (ap \ c\_2Ebool\_2E\_2F\_5C \ (ap \ (ap \ (c\_2Ebool\_2EIN \\
& A\_27a) \ V3x) \ V1s)) \ (ap \ V0P \ V3x))))))))) \quad (5)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow \forall A\_27c. \\
& nonempty \ A\_27c \Rightarrow (\forall V0P \in A\_27a.(\forall V1s \in (2^{A\_27b}).( \\
& \forall V2x \in A\_27c.((p \ (ap \ (ap \ (c\_2Ebool\_2ERES\_EXISTS\_UNIQUE \\
& A\_27b) \ V1s) \ (\lambda V3x \in A\_27b.c\_2Ebool\_2ET)))) \Leftrightarrow (p \ (ap \ (c\_2Ebool\_2E\_3F\_21 \\
& A\_27b) \ (\lambda V4x \in A\_27b.(ap \ (ap \ (c\_2Ebool\_2EIN \ A\_27b) \ V4x) \ V1s)))))))))
\end{aligned}$$