

thm_2Eres__quan_2ERES__FORALL__CONJ__DIST (TMGvW67W2qUfbVsRm9Lai3kEduepsvuNHXc)

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Definition 1 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_}o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_}o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a})) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)))$

Definition 5 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t)))$

Definition 6 We define `c_2Ebool_2E_IN` to be $\lambda A_27a : \iota. (\lambda V0f \in A_27a. (\lambda V1f \in (2^{A_27a}). (\text{ap } V1f V0f)))$

Definition 7 We define `c_2Ebool_2ERES__FORALL` to be $\lambda A_27a : \iota. (\lambda V0p \in (2^{A_27a}). (\lambda V1m \in (2^{A_27a}). (\text{ap } V1m V0p)))$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (1)$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1f \in (2^{A_27a}). ((p (\text{ap } (\text{ap } (\text{c_2Ebool_2ERES__FORALL } A_27a) V0P) V1f)) \Leftrightarrow (\forall V2x \in A_27a. ((p (\text{ap } (\text{ap } (\text{c_2Ebool_2EIN } A_27a) V2x) V0P)) \Rightarrow (p (\text{ap } V1f V2x))))))) \quad (2)$$

Theorem 1

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). (\forall V2R \in (2^{A_27a}). ((p (\text{ap } (\text{ap } (\text{c_2Ebool_2ERES__FORALL } A_27a) V0P) (\lambda V3i \in A_27a. (\text{ap } (\text{ap } (\text{c_2Ebool_2E_2F_5C } (\text{ap } V1Q V3i)) (\text{ap } V2R V3i)))))) \Leftrightarrow ((p (\text{ap } (\text{ap } (\text{c_2Ebool_2ERES__FORALL } A_27a) V0P) (\lambda V4i \in A_27a. (\text{ap } V1Q V4i)))) \wedge (p (\text{ap } (\text{ap } (\text{c_2Ebool_2ERES__FORALL } A_27a) V0P) (\lambda V5i \in A_27a. (\text{ap } V2R V5i))))))))))$$