# thm_2Eres__quan_2ERES__FORALL__DISJ__DIST (TMFg8P4BQkujmcE1PaKtSiPRXvmx2MeRYfE)

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**Definition 1** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 2** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 3** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

**Definition 4** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

**Definition 5** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

**Definition 6** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

**Definition 7** *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

**Definition 8** *We define* c_2Ebool_2ERES__FORALL *to be* $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).(a$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{1}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ V0P)) \Leftrightarrow (p\ (ap\ V0P\ V1x))))) \tag{2}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1f \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ V0P)\ V1f)) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0P)) \Rightarrow (p\ (ap\ V1f\ V2x))))))) \tag{3}$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).(\forall V2R \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (\lambda V3j \in A\_27a.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_5C\_2F}\ (ap\ V0P\ V3j))\ (ap\ V1Q\ V3j))))\ (\lambda V4i \in A\_27a.(ap\ V2R\ V4i)))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ V0P)\ (\lambda V5i \in A\_27a.(ap\ V2R\ V5i)))) \land (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ V1Q)\ (\lambda V6i \in A\_27a.(ap\ V2R\ V6i)))))))))$$