

thm_2Eres__quan_2ERES__FORALL__FORALL
 (TMRuTqXh-
 WyZ6v6wWBU9jbNc3PGNoB5YQbuh)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define c_2Ebool_2EET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)))))$

Definition 6 We define $c_2Ebool_2ERES_FORALL$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)))))$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (1)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V1x) V0P)) \Leftrightarrow (p (ap V0P V1x)))))) \quad (2)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1f \in (2^{A_27a}).((p (ap (ap (c_2Ebool_2ERES_FORALL A_27a) V0P) V1f)) \Leftrightarrow (\forall V2x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V0P)) \Rightarrow (p (ap V1f V2x)))))))) \quad (3)$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\\ & \quad \forall V0P \in (2^{A_{27a}}).(\forall V1R \in ((2^{A_{27b}})^{A_{27a}}).(\forall V2x \in \\ & \quad A_{27b}.((\forall V3x \in A_{27b}.(p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\ & \quad A_{27a})\ V0P)\ (\lambda V4i \in A_{27a}.(ap\ (ap\ V1R\ V4i)\ V3x)))))) \Leftrightarrow (p\ (ap\ (ap \\ & \quad (c_2Ebool_2ERES_FORALL\ A_{27a})\ V0P)\ (\lambda V5i \in A_{27a}.(ap\ (c_2Ebool_2E.21 \\ & \quad A_{27b})\ (\lambda V6x \in A_{27b}.(ap\ (ap\ V1R\ V5i)\ V6x)))))))))) \end{aligned}$$