

thm\_2Eres\_\_quan\_2ERES\_SELECT  
(TMJejKiXPJUSeUQA9BrLXLk8QaSNdnJVS53)

October 26, 2020

**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2EIN` to be  $\lambda A_{.27a} : \iota.(\lambda V0x \in A_{.27a}.\lambda V1f \in (2^{A_{.27a}}).\lambda V2x \in A_{.27a}.(ap\ V1f\ V2x))$

**Definition 3** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 4** We define `c_2Ebool_2EET` to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define `c_2Ebool_2E_21` to be  $\lambda A_{.27a} : \iota.(\lambda V0P \in (2^{A_{.27a}}).\lambda V1f \in (2^{A_{.27a}}).\lambda V2x \in A_{.27a}.(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A_{.27a}}))\ V2x))\ V0P))$

**Definition 6** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))\ V0t1))$

**Definition 7** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A \wedge p\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define `c_2Ebool_2ERES_SELECT` to be  $\lambda A_{.27a} : \iota.(\lambda V0p \in (2^{A_{.27a}}).\lambda V1m \in (2^{A_{.27a}}).\lambda V2x \in A_{.27a}.(ap\ (ap\ (c\_2Emin\_2E\_40\ A_{.27a})\ (\lambda V3x \in A_{.27a}.(ap\ (ap\ (c\_2Ebool\_2E\_2F\_5C\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A_{.27a})\ V3x)\ V0p))\ V1m))\ V2x))\ V0p))$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).\forall V1f \in \\ & (2^{A_{.27a}}).\lambda V2x \in A_{.27a}.(ap\ (ap\ (c\_2Ebool\_2ERES\_SELECT\ A_{.27a})\ V0P)\ V1f) = \\ & (ap\ (c\_2Emin\_2E\_40\ A_{.27a})\ (\lambda V3x \in A_{.27a}.(ap\ (ap\ c\_2Ebool\_2E\_2F\_5C \\ & (ap\ (ap\ (c\_2Ebool\_2EIN\ A_{.27a})\ V3x)\ V0P))\ V1f\ V2x)))))) \end{aligned} \quad (1)$$

**Theorem 1**

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).\forall V1f \in \\ & (2^{A_{.27a}}).\lambda V2x \in A_{.27a}.(ap\ (ap\ (c\_2Ebool\_2ERES\_SELECT\ A_{.27a})\ V0P)\ V1f) = \\ & (ap\ (c\_2Emin\_2E\_40\ A_{.27a})\ (\lambda V3x \in A_{.27a}.(ap\ (ap\ c\_2Ebool\_2E\_2F\_5C \\ & (ap\ (ap\ (c\_2Ebool\_2EIN\ A_{.27a})\ V3x)\ V0P))\ V1f\ V2x)))))) \end{aligned}$$