

thm\_2Erich\_list\_2EAND\_EL\_FOLDL  
(TMJSE4j82EdjKiriBGYj3M4aWMXABe5XTTh)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.(\lambda V0x \in A.\lambda V1y \in A.V0x)$

**Definition 3** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A.\lambda V1c \in A.V0f))^{A.\lambda V2c \in A.V1c})^{A.\lambda V3c \in A.V2c}$

**Definition 4** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A.\lambda a : \iota.(ap (ap (c\_2Ecombin\_2ES A.\lambda V1a (A.\lambda V2a^{A.\lambda V3a})) A.\lambda V4a))^{A.\lambda V5a}$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2EEVERY : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.\lambda a.nonempty A \Rightarrow c\_2Elist\_2EEVERY A \in ((2^{(ty\_2Elist\_2Elist A)})^{(2^{A.\lambda V1a})})^{(2^{A.\lambda V2a})} \quad (2)$$

**Definition 5** We define  $c\_2Erich\_list\_2EAND\_EL$  to be  $(ap (c\_2Elist\_2EEVERY 2) (c\_2Ecombin\_2EI 2))$ .

**Definition 6** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A.\lambda V1a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A.\lambda V2a})) (\lambda V3x \in 2.V3x)) (\lambda V4x \in 2.V4x)))^{A.\lambda V5a}$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))^{A.\lambda V3t}$

Let  $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.\lambda a.nonempty A \Rightarrow \forall A.\lambda b.nonempty A \Rightarrow c\_2Elist\_2EFOLDL A.\lambda V1a A.\lambda V2b \in (((A.\lambda V3b^{(ty\_2Elist\_2Elist A.\lambda V4a)})^{A.\lambda V5b})^{(A.\lambda V6b^{A.\lambda V7a})})^{A.\lambda V8b} \quad (3)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. ((ap\ (c.2Ecombin.2E\ A.27a)\ V0x) = V0x)) \quad (4)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1l \in \\ (ty.2Elist.2Elist\ A.27a). ((p\ (ap\ (ap\ (c.2Elist.2EEVERY\ A.27a)\ \\ V0P)\ V1l)) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (c.2Elist.2EFOLDL\ A.27a\ 2)\ (\lambda V2l.27 \in \\ 2.(\lambda V3x \in A.27a.(ap\ (ap\ c.2Ebool.2E.2F.5C\ V2l.27)\ (ap\ V0P\ V3x)))) \\ c.2Ebool.2ET)\ V1l)))))) \end{aligned} \quad (5)$$

**Theorem 1**

$$\begin{aligned} (\forall V0l \in (ty.2Elist.2Elist\ 2). ((p\ (ap\ c.2Erich\_list.2EAND\_EL \\ V0l)) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (c.2Elist.2EFOLDL\ 2\ 2)\ c.2Ebool.2E.2F.5C) \\ c.2Ebool.2ET)\ V0l)))) \end{aligned}$$