

# thm\_Erich\_list\_EAPPEND\_FOLDR (TMVL- rmYi29wMMaQ4uP5Tpfc1geg6HFy5hLJ)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EAPPEND A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{(ty\_2Elist\_2Elist A\_27a)}) \quad (2)$$

Let  $c\_2Elist\_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Elist\_2EFOLDR A\_27a A\_27b \in (((A\_27b)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27b})^{((A\_27b)^{A\_27b})^{A\_27a}} \quad (3)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (4)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (5)$$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow p Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{8}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & ((\forall V0l \in (ty\_2Elist\_2Elist \\ & A\_27a).((ap (ap (c\_2Elist\_2EAPPEND A\_27a) (c\_2Elist\_2ENIL A\_27a)) \\ & V0l) = V0l)) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist A\_27a).(\forall V2l2 \in \\ & (ty\_2Elist\_2Elist A\_27a).(\forall V3h \in A\_27a.((ap (ap (c\_2Elist\_2EAPPEND \\ & A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V3h) V1l1)) V2l2) = (ap (ap \\ & (c\_2Elist\_2ECONS A\_27a) V3h) (ap (ap (c\_2Elist\_2EAPPEND A\_27a) \\ & V1l1) V2l2)))))))))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & (\forall V0f \in ((A\_27b^{A\_27b})^{A\_27a}).(\forall V1e \in A\_27b.((ap ( \\ & ap (ap (c\_2Elist\_2EFOLDR A\_27a A\_27b) V0f) V1e) (c\_2Elist\_2ENIL \\ & A\_27a)) = V1e))) \wedge (\forall V2f \in ((A\_27b^{A\_27b})^{A\_27a}).(\forall V3e \in \\ & A\_27b.(\forall V4x \in A\_27a.(\forall V5l \in (ty\_2Elist\_2Elist A\_27a). \\ & ((ap (ap (ap (c\_2Elist\_2EFOLDR A\_27a A\_27b) V2f) V3e) (ap (ap (c\_2Elist\_2ECONS \\ & A\_27a) V4x) V5l)) = (ap (ap V2f V4x) (ap (ap (ap (c\_2Elist\_2EFOLDR \\ & A\_27a A\_27b) V2f) V3e) V5l)))))))))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & (\forall V0P \in (2^{(ty\_2Elist\_2Elist A\_27a)}). \\ & (((p (ap V0P (c\_2Elist\_2ENIL A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\ & A\_27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A\_27a.(p (ap V0P (ap (ap ( \\ & c\_2Elist\_2ECONS A\_27a) V2h) V1t))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ & A\_27a).(p (ap V0P V3l)))))) \end{aligned} \tag{11}$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & (\forall V0l1 \in (ty\_2Elist\_2Elist \\ & A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist A\_27a).((ap (ap (c\_2Elist\_2EAPPEND \\ & A\_27a) V0l1) V1l2) = (ap (ap (ap (c\_2Elist\_2EFOLDR A\_27a (ty\_2Elist\_2Elist \\ & A\_27a)) (c\_2Elist\_2ECONS A\_27a) V1l2) V0l1)))))) \end{aligned}$$