thm_2Erich__list_2EASSOC__FOLDL__FLAT (TMUrZ4b9XwtfGyEF88MLsALYqRuBD2bTtRq)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ecombin_2EASSOC *to be* $\lambda A\_27a : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool}$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{2}$$

Let $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EMAP\ A\_27a\ A\_27b \in (((ty\_2Elist\_2Elist\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{3}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{4}$$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ESNOC\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (5)$$

Let $c\_2Elist\_2EFLAT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EFLAT\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ (ty\_2Elist\_2Elist\ A\_27a))}) \quad (6)$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (7)$$

Let $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b^{A\_27a})^{A\_27b})}) \quad (8)$$

Definition 7 *We define* c_2Ecombin_2ERIGHT__ID *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27b})^{A\_27a})$.

Definition 8 *We define* c_2Ecombin_2EFCOMM *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27a^{A\_27}$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in ((A\_27a^{A\_27a})^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EFCOMM}\ A\_27a\ A\_27a\ A\_27a)\ V0f)\ V0f)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Ecombin\_2EASSOC}\ A\_27a)\ V0f)))) \quad (13)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elist\_2EFLAT\ A\_27a)\ (c\_2Elist\_2ENIL\ (ty\_2Elist\_2Elist\ A\_27a))) = (c\_2Elist\_2ENIL\ A\_27a)) \wedge (\forall V0h \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1t \in (ty\_2Elist\_2Elist\ (ty\_2Elist\_2Elist\ A\_27a)).((ap\ (c\_2Elist\_2EFLAT\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Elist\_2Elist\ A\_27a))\ V0h)\ V1t)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0h)\ (ap\ (c\_2Elist\_2EFLAT\ A\_27a)\ V1t))))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V0f)\ (c\_2Elist\_2ENIL\ A\_27a)) = (c\_2Elist\_2ENIL\ A\_27b))) \wedge (\forall V1f \in (A\_27b^{A\_27a}).(\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V1f)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ (ap\ V1f\ V2h))\ (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V1f)\ V3t))))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V0f)\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V1x)\ V2l)) = (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27b)\ (ap\ V0f\ V1x))\ (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V0f)\ V2l)))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).(((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1l)) \Rightarrow (\forall V2x \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V2x)\ V1l))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l)))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((A\_27b^{A\_27a})^{A\_27b}).(\forall V1e \in A\_27b.(\forall V2x \in A\_27a.(\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V0f)\ V1e)\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V2x)\ V3l)) = (ap\ (ap\ V0f\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V0f)\ V1e)\ V3l))\ V2x)))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l \in (ty\_2Elist\_2Elist\ (ty\_2Elist\_2Elist\ A\_27a)).((ap\ (c\_2Elist\_2EFLAT\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ (ty\_2Elist\_2Elist\ A\_27a))\ V0x)\ V1l)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (c\_2Elist\_2EFLAT\ A\_27a)\ V1l))\ V0x)))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((A\_27a^{A\_27b})^{A\_27a}).(\forall V1g \in ((A\_27a^{A\_27a})^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EFCOMM}\ A\_27a\ A\_27b\ A\_27a)\ V0f)\ V1g)) \Rightarrow (\forall V2e \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2ERIGHT\_\_ID}\ A\_27a\ A\_27a)\ V1g)\ V2e)) \Rightarrow (\forall V3l1 \in (ty\_2Elist\_2Elist\ A\_27b).(\forall V4l2 \in (ty\_2Elist\_2Elist\ A\_27b).((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27b\ A\_27a)\ V0f)\ V2e)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27b)\ V3l1)\ V4l2)) = (ap\ (ap\ V1g\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27b\ A\_27a)\ V0f)\ V2e)\ V3l1))\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27b\ A\_27a)\ V0f)\ V2e)\ V4l2)))))))))) \tag{20}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in ((A\_27a^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Ecombin\_2EASSOC}\ A\_27a)\ V0f)) \Rightarrow (\forall V1e \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2ERIGHT\_\_ID}\ A\_27a\ A\_27a)\ V0f)\ V1e)) \Rightarrow (\forall V2l \in (ty\_2Elist\_2Elist\ (ty\_2Elist\_2Elist\ A\_27a)).((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27a)\ V0f)\ V1e)\ (ap\ (c\_2Elist\_2EFLAT\ A\_27a)\ V2l)) = (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27a)\ V0f)\ V1e)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ (ty\_2Elist\_2Elist\ A\_27a)\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27a)\ V0f)\ V1e))\ V2l))))))))$$