

thm_2Erich__list_2EASSOC__FOLDR__FLAT
(TMSXLUAKn-
RofFpR8n6B6BtBMVmQEwZb1gFH)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ecombin_2EASSOC$ to be $\lambda A_27a : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27a}).(ap (c_2Ebool$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EFLAT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EFLAT A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist (ty_2Elist_2Elist A_27a))}) \quad (2)$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EMAP A_27a A_27b \in (((ty_2Elist_2Elist A_27b)^{(ty_2Elist_2Elist A_27a)})^{(A_27b^{A_27a})}) \quad (3)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (5)$$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)(ty_2Elist_2Elist\ A_27a))(ty_2Elist_2Elist\ A_27a)) \quad (6)$$

Let $c_2Elist_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDR\ A_27a\ A_27b \in (((A_27b^{(ty_2Elist_2Elist\ A_27a)})_{A_27b})_{((A_27b^{A_27b})^{A_27a})}) \quad (7)$$

Definition 9 We define $c_2Ecombin_2ELEFT_ID$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27b^{A_27b})^{A_27a}).\lambda$

Definition 10 We define $c_2Ecombin_2EFCOMM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b})^{A_27c}$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in ((A_27a^{A_27a})^{A_27a}).((p\ (ap\ (ap\ (c_2Ecombin_2EFCOMM\ A_27a\ A_27a\ A_27a)\ V0f)) \Leftrightarrow (p\ (ap\ (c_2Ecombin_2EASSOC\ A_27a)\ V0f)))) \quad (13)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Elist_2EFLAT\ A_27a)\ (\\
& \quad c_2Elist_2ENIL\ (ty_2Elist_2Elist\ A_27a))) = (c_2Elist_2ENIL \\
& \quad A_27a)) \wedge (\forall V0h \in (ty_2Elist_2Elist\ A_27a). (\forall V1t \in \\
& \quad (ty_2Elist_2Elist\ (ty_2Elist_2Elist\ A_27a)). ((ap\ (c_2Elist_2EFLAT \\
& \quad A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Elist_2Elist\ A_27a)\ V0h) \\
& \quad V1t)) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0h)\ (ap\ (c_2Elist_2EFLAT \\
& \quad A_27a)\ V1t))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad (\forall V0f \in (A_27b^{A_27a}). ((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b) \\
& \quad V0f)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27b))) \wedge (\forall V1f \in \\
& \quad (A_27b^{A_27a}). (\forall V2h \in A_27a. (\forall V3t \in (ty_2Elist_2Elist \\
& \quad A_27a). ((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ (ap\ V1f\ V2h)) \\
& \quad (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V3t))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad (\forall V0f \in ((A_27b^{A_27b})^{A_27a}). (\forall V1e \in A_27b. ((ap\ (\\
& \quad ap\ (ap\ (c_2Elist_2EFOLDR\ A_27a\ A_27b)\ V0f)\ V1e)\ (c_2Elist_2ENIL \\
& \quad A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b^{A_27b})^{A_27a}). (\forall V3e \in \\
& \quad A_27b. (\forall V4x \in A_27a. (\forall V5l \in (ty_2Elist_2Elist\ A_27a). \\
& \quad ((ap\ (ap\ (ap\ (c_2Elist_2EFOLDR\ A_27a\ A_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_27a)\ V4x)\ V5l)) = (ap\ (ap\ V2f\ V4x)\ (ap\ (ap\ (ap\ (c_2Elist_2EFOLDR \\
& \quad A_27a\ A_27b)\ V2f)\ V3e)\ V5l))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\
& \quad (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& \quad A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\
& \quad c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& \quad A_27a). (p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0g \in ((A.27a^{A.27a})^{A.27a}).(\forall V1f \in ((A.27a^{A.27a})^{A.27b}). \\
& \quad ((p\ (ap\ (ap\ (c.2Ecombin_2EFCOMM\ A.27a\ A.27a\ A.27b)\ V0g)\ V1f)) \Rightarrow (\\
& \quad \quad \forall V2e \in A.27a.((p\ (ap\ (ap\ (c.2Ecombin_2ELEFT_ID\ A.27a\ A.27a) \\
& \quad \quad \quad V0g)\ V2e)) \Rightarrow (\forall V3l1 \in (ty_2Elist_2Elist\ A.27b).(\forall V4l2 \in \\
& \quad \quad \quad (ty_2Elist_2Elist\ A.27b).((ap\ (ap\ (ap\ (c.2Elist_2EFOLDR\ A.27b \\
& \quad \quad \quad A.27a)\ V1f)\ V2e)\ (ap\ (ap\ (c.2Elist_2EAPPEND\ A.27b)\ V3l1)\ V4l2)) = \\
& \quad \quad \quad (ap\ (ap\ V0g\ (ap\ (ap\ (ap\ (c.2Elist_2EFOLDR\ A.27b\ A.27a)\ V1f)\ V2e)\ V3l1)) \\
& \quad \quad \quad (ap\ (ap\ (ap\ (c.2Elist_2EFOLDR\ A.27b\ A.27a)\ V1f)\ V2e)\ V4l2))))))))) \\
& \quad \quad \quad (18)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in ((A.27a^{A.27a})^{A.27a}). \\
& \quad ((p\ (ap\ (c.2Ecombin_2EASSOC\ A.27a)\ V0f)) \Rightarrow (\forall V1e \in A.27a. \\
& \quad ((p\ (ap\ (ap\ (c.2Ecombin_2ELEFT_ID\ A.27a\ A.27a)\ V0f)\ V1e)) \Rightarrow (\forall V2l \in \\
& \quad (ty_2Elist_2Elist\ (ty_2Elist_2Elist\ A.27a)).((ap\ (ap\ (ap\ (c.2Elist_2EFOLDR \\
& \quad \quad A.27a\ A.27a)\ V0f)\ V1e)\ (ap\ (c.2Elist_2EFLAT\ A.27a)\ V2l)) = (ap\ (ap \\
& \quad (ap\ (c.2Elist_2EFOLDR\ A.27a\ A.27a)\ V0f)\ V1e)\ (ap\ (ap\ (c.2Elist_2EMAP \\
& \quad \quad (ty_2Elist_2Elist\ A.27a)\ A.27a)\ (ap\ (ap\ (c.2Elist_2EFOLDR\ A.27a \\
& \quad \quad \quad A.27a)\ V0f)\ V1e))\ V2l)))))))))
\end{aligned}$$