

# thm\_2Erich\_\_list\_2EBUTLASTN\_\_1 (TM- Pzru6h9e1XmvsyCkYSGTmWAFdrvEPX9Z7)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 7** We define  $c\_2Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmic\_2EZERO) V0n)$ .

**Definition 8** We define  $c\_2Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21) 2) (\lambda V0t \in 2.V0t)$ .

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21))$ .

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (7)$$

Let  $c\_2Elist\_2EFRONT : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EFRONT A\_27a \in ((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)}) \quad (8)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (9)$$

Let  $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ESNOC A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (10)$$

Let  $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EREVERSE A\_27a \in ((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)}) \quad (11)$$

Let  $c\_2Elist\_2EDROP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EDROP A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 12** We define  $c\_2Erich\_list\_2EBUTLASTN$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1xs$

**Definition 13** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t \in 2.V2t)))$

Assume the following.

$$((ap c\_2Earithmic\_2ENUMERAL (ap c\_2Earithmic\_2EBIT1 c\_2Earithmic\_2EZERO)) = (ap c\_2Enum\_2ESUC c\_2Enum\_2E0)) \quad (13)$$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (16)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1l \in \\ & (ty\_2Elist\_2Elist A.27a).((ap (c\_2Elist\_2EFront A.27a) (ap ( \\ & (c\_2Elist\_2ESNOC A.27a) V0x) V1l)) = V1l))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist A.27a)}), \\ & (((p (ap V0P (c\_2Elist\_2ENIL A.27a))) \wedge (\forall V1l \in (ty\_2Elist\_2Elist \\ & A.27a).((p (ap V0P V1l)) \Rightarrow (\forall V2x \in A.27a.(p (ap V0P (ap (ap ( \\ & c\_2Elist\_2ESNOC A.27a) V2x) V1l)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ & A.27a).(p (ap V0P V3l)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ & (\forall V0l \in (ty\_2Elist\_2Elist A.27a).((ap (ap (c\_2Erich\_list\_2EBUTLASTN \\ & A.27a) c\_2Enum\_2E0) V0l) = V0l) \wedge (\forall V1n \in ty\_2Enum\_2Enum. \\ & (\forall V2x \in A.27b.(\forall V3l \in (ty\_2Elist\_2Elist A.27b).( \\ & (ap (ap (c\_2Erich\_list\_2EBUTLASTN A.27b) (ap c\_2Enum\_2ESUC V1n)) \\ & (ap (ap (c\_2Elist\_2ESNOC A.27b) V2x) V3l)) = (ap (ap (c\_2Erich\_list\_2EBUTLASTN \\ & A.27b) V1n) V3l)))))) \end{aligned} \quad (20)$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ & A.27a).((\neg(V0l = (c\_2Elist\_2ENIL A.27a))) \Rightarrow ((ap (ap (c\_2Erich\_list\_2EBUTLASTN \\ & A.27a) (ap c\_2Earithmic\_2ENUMERAL (ap c\_2Earithmic\_2EBIT1 \\ & c\_2Earithmic\_2EZERO))) V0l) = (ap (c\_2Elist\_2EFront A.27a) \\ & V0l)))) \end{aligned}$$